87th Congress }
1st Session } JOINT COMMITTEE PRINT

INVENTORY FLUCTUATIONS AND ECONOMIC STABILIZATION

MATERIALS PREPARED FOR THE

JOINT ECONOMIC COMMITTEE
CONGRESS OF THE UNITED STATES

PART II

CAUSATIVE FACTORS IN MOVEMENTS OF BUSINESS INVENTORIES

Printed for the use of the Joint Economic Committee

U.S. GOVERNMENT PRINTING OFFICE
76626 WASHINGTON : 1961

For sale by the Superintendent of Documents, U.S. Government Printing Office
Washington 25, D.C. - Price 55 cents

JOINT ECONOMIC COMMITTEE

(Created pursuant to sec. 5(a) of Public Law 304, 79th Cong.)

WRIGHT PATMAN, Texas, *Chairman*
PAUL H. DOUGLAS, Illinois, *Vice Chairman*

HOUSE OF REPRESENTATIVES	SENATE
RICHARD BOLLING, Missouri	JOHN SPARKMAN, Alabama
HALE BOGGS, Louisiana	J. W. FULBRIGHT, Arkansas
HENRY S. REUSS, Wisconsin	WILLIAM PROXMIRE, Wisconsin
MARTHA W. GRIFFITHS, Michigan	CLAIBORNE PELL, Rhode Island
THOMAS B. CURTIS, Missouri	PRESCOTT BUSH, Connecticut
CLARENCE E. KILBURN, New York	JOHN MARSHALL BUTLER, Maryland
WILLIAM B. WIDNALL, New Jersey	JACOB K. JAVITS, New York

WM. SUMMERS JOHNSON, *Executive Director*
JOHN W. LEHMAN, *Deputy Executive Director*
RICHARD J. BARBER, *Clerk*

LETTERS OF TRANSMITTAL

DECEMBER 21, 1961.

To Members of the Joint Economic Committee:

Transmitted herewith for the use of the Joint Economic Committee and other Members of the Congress is part II of a three-part series of papers prepared by experts from government, the colleges, and research organizations. These materials have been assembled under the general title of "Inventory Fluctuations and Economic Stabilization."

The papers contained in this and the other two volumes will be discussed by their authors and other experts in a series of public hearings to be held early next year.

The papers have been prepared and the hearings are being arranged in accordance with the program of work set forth in the committee's annual report filed with the Congress May 2, 1961 (H. Rept. 328, 87th Cong., 1st sess., p. 47). This program provides for a "study of inventory movements, accumulation and liquidation" in the following language:

> Inventory fluctuation and behavior will be studied to try to determine the extent to which changes in inventories are causes of instability and to what extent they are in themselves affected by other forces inherent in the business cycle. The committee will be concerned with such areas as merchandising and production planning to see what influences and what can be done to regularize purchasing so that characteristically wide swings in the direction of inventory adjustments can be minimized.

The three sets of reports being transmitted are limited to the fact-finding phase of the study outlined in the program.

The papers are presented in advance of the committee's hearings in accordance with a Joint Economic Committee practice of providing members of the committee and participating panelists an opportunity, whenever possible, to examine thoroughly the analyses and findings in preparation for the discussions at the hearings.

Sincerely yours,

WRIGHT PATMAN, *Chairman.*

DECEMBER 19, 1961.

Hon. WRIGHT PATMAN,
Chairman, Joint Economic Committee,
House of Representatives, Washington, D.C.

DEAR MR. CHAIRMAN: Transmitted herewith is part II of a three-part series of papers on the general subject of "Inventory Fluctuations and Economic Stabilization." The two papers in part I are devoted principally to a descriptive analysis of postwar inventory fluctuations. The papers in part II deal with the causative factors in movements in business inventories and the papers in part III with the relationship between inventory movements and economic instability. There are also included in part III a bibliography and a paper concerned with the availability and reliability of statistical data on inventories.

Professor Paul G. Darling, on loan to the committee from Bowdoin College, has had major staff responsibility for formulating and directing this study.

Sincerely yours,

WM. SUMMERS JOHNSON, *Executive Director.*

CONTENTS

FIRM COST STRUCTURES AND THE DYNAMIC RESPONSES OF INVENTORIES, PRODUCTION, WORK FORCE, AND ORDERS TO SALES FLUCTUATIONS, BY CHARLES C. HOLT, SOCIAL SYSTEMS RESEARCH INSTITUTE, UNIVERSITY OF WISCONSIN, AND FRANCO MODIGLIANI, NORTHWESTERN UNIVERSITY

CHANGES IN OWNERSHIP OF PURCHASED MATERIALS, BY RUTH P. MACK, NATIONAL BUREAU OF ECONOMIC RESEARCH

MEASURES OF INVENTORY CONDITIONS, BY NESTOR E. TERLECKYJ, ASSISTED BY ALFRED TELLA, NATIONAL BUREAU OF ECONOMIC RESEARCH

CHARTS

TABLES

FIRM COST STRUCTURES AND THE DYNAMIC RESPONSES OF INVENTORIES, PRODUCTION, WORK FORCE, AND ORDERS TO SALES FLUCTUATIONS

BY

CHARLES C. HOLT
Social Systems Research Institute
University of Wisconsin

AND

FRANCO MODIGLIANI
Northwestern University

FIRM COST STRUCTURES AND THE DYNAMIC RESPONSES OF INVENTORIES, PRODUCTION, WORK FORCE, AND ORDERS TO SALES FLUCTUATIONS

Overview of the Paper

In studying the inventory fluctuations of the economy, economists have thought that businessmen endeavored to maintain certain relationships between their sales and their inventories. Study of the data on the economy and industry levels has in general supported these views, but we have lacked sufficiently precise notions of what these relationships are and why business firms behave as they do.

In recent years mathematical methods have been developed for making production and inventory control decisions that are best in the sense of minimizing costs. Some companies have used them to improve their decision performances but they can also be used by economists to predict the actions that business managers using judgment would tend to take when they face certain cost and profit considerations. Used in this way such analyses can throw a great deal of light on the complex dynamic behavior of business firms when they are faced with fluctuating sales and uncertainty about future business conditions. Applying the decision analysis associated with quadratic cost functions to certain decision problems related to production and inventories the following conclusions are reached in this paper and are presented briefly in this summary without necessary qualifications:

(*a*) The general conclusions which economists have reached, that inventories tend to fluctuate with sales and production in such a way as to aggravate economic fluctuations, is corroborated by the study of the cost considerations affecting production, employment, and inventory decisions in the firm. Although erratic and excessive responses by firms to changes in sales undoubtedly contribute to fluctuations, the *systematic* tendency of inventories to contribute to fluctuations can be traced to the rational efforts of business firms to keep their costs down and their profits up.

(*b*) When the level of sales and production rises, factories, wholesalers, and retailers need more purchased materials inventories, in-process inventories, and finished goods inventories if they are to keep certain costs and penalties down. Specifically when sales are high, shipping costs and machine setups are reduced by increasing shipping and production lots, and this requires more inventories. Also when sales are high, unexpected fluctuations in the demands for particular items are increased, and this requires additional inventories to serve as buffers if the production disruption of stock outs and the cost of lost sales are to be avoided. Economists have recognized for some time that as a consequence of these cost considerations there is a strong tendency for inventories to be high when sales are high and low when sales are low.

(*c*) But there are other costs to be considered as well. Because of overtime, hiring, and layoff costs, etc., it is costly for a manufacturer to fluctuate his production rate or the size of his work force so he would like if possible to smooth production.

(*d*) However, when sales fluctuate with general business conditions or with industry conditions, the manufacturer has no choice but to incur the costs either of production fluctuations or of deviations of finished goods inventories or the backlog of unfilled orders from their most economical levels. Given the necessity of responding to fluctuations in sales and orders the manufacturer can be expected in the interests of keeping his costs down to smooth his production but not so much as to allow excessive swings in the inventory and backlog positions from their economical levels. This is done by trying to forecast sales and scheduling somewhat smoothed production on the basis of sales anticipated for some months ahead and corresponding desired changes in the inventory position. Inevitably the finished inventory-backlog position will depart from the desired levels as the result of forecast errors, and production is adjusted to achieve a gradual correction spread over several months. How far ahead the forecasts are made and how fast the inventory adjustments are made depends on the cost structure of the particular firm and industry.

(*e*) Thus from (*b*), (*c*), and (*d*) we see that when factory sales fluctuate there are two counteracting tendencies based on cost considerations. The desire to have higher inventories when sales increase tends to make production build up in anticipation of an increase in sales and actually fluctuate even more than sales, and working in the opposite direction, efforts to avoid production fluctuations tend to smooth production and work force so that they fluctuate less than sales. In addition, forecasts of changes in sales often lag behind the actual change with the result that the production for the inventory buildup tends to coincide with the sales fluctuation instead of leading it. Considering the above points, the overall response of production will depend upon the cost structure and forecast performance of the firm.

It seems clear that the firms in a great many industries will have cost structures so that when facing an uncertain cyclic sales fluctuation of 2 to 3 years they will fluctuate their production in almost exact synchronism with their sales but with a somewhat larger amplitude so that they are actually amplifying the sales fluctuations.

(*f*) The tendency for sales forecasts to lag contributes further to the amplification tendency.

(*g*) Because production fluctuations can be accommodated economically with overtime and slack time, the work force will tend to fluctuate less than production.

(*h*) Purchased materials inventories of a factory can be adjusted by placing orders on suppliers, and since there are normally no cost penalties for placing fluctuating orders, these inventories will tend to be adjusted quickly *without any smoothing*. The result is that an amplification of fluctuations can be expected in the placement of orders especially if the lead times are long.

(*i*) For exactly the same reasons retailers and wholesalers will tend in their placement of orders to amplify fluctuations in their sales.

(*j*) Since orders often pass through a whole chain of suppliers and manufacturers, any tendencies toward amplification of fluctuations will tend to have a cumulative effect.

(*k*) In addition to the well known social effects of economic fluctuations, private costs occur as well. By the use of cost functions and decision analyses, estimates of the costs of economic fluctuations to business firms can be made.

(*l*) Qualitative conclusions of the type reached in this paper are only a beginning step. To improve our understanding of inventory dynamics there is a great need for quantitative empirical research relating to industry cost structures, decision responses, and forecast dynamics. The approach developed in this paper utilizing optimizing decision analyses should be helpful in directing attention toward the critical relationships.

(*m*) The implications of the dynamic relationships considered in this study for the stability of the economy are by no means obvious since stability is determined by the interaction between these relations and those in the rest of the economy. Additional research is badly needed on this level.

I. Nature of the Problem and Method of Attack

The role of inventory investments and disinvestments as a possible and probable source of periodic fluctuations in aggregate economic activity has long been recognized and has received considerable attention in recent years. Our understanding of this phenomenon has been greatly advanced through the construction of inventory cycle models, especially since the pioneering contributions of Lloyd Metzler [22], [23].[1] One of the basic building blocks in such models is represented by a set of hypotheses concerning production and inventory decisions by business firms. Unfortunately the hypotheses that have been utilized so far have been generally of a broad aggregative type and relatively little effort has been made to relate or ground these macroeconomic assumptions on an analysis of microeconomic behavior. This failure can be traced, at least in part, to a lack of tools suitable for a microeconomic analysis of the type required.

Fortunately in the last few years there has been a considerable amount of fundamental research on optimal inventory control and optimal production and employment scheduling. This research has provided practicable decision analyses for solving a number of problems in this area and has also considerably increased our understanding of actual (as distinguished from optimal) practices in inventory control and production scheduling. The objective of the authors is to exploit these recent advances in an effort to build a bridge between macroeconomic assumptions and microeconomic analysis. This paper concentrates on developing the microeconomic foundations of the analysis hoping to contribute thereby also to the development of a dynamic theory of the firm. A sequel paper to be published elsewhere will then proceed to develop the macrodynamic implications of the analysis. We hope to be able to show that microeconomic analysis broadly supports the work done by Metzler and others. At the same time it promises to enrich and deepen our understanding of the nature and character of the inventory cycle and to contribute to the solution of certain dynamic aspects of the aggregation problem.

The road on which we embark is not entirely uncharted. Among the earlier contributions, we wish to call particular attention to the work of Ruth Mack, [19], [20], [21], who has repeatedly emphasized the importance of linking macroeconomic hypotheses to analyses of microeconomic behavior and to the work of T. Whitin, [37], Mills, [25], [26], and Modigliani [27], who have made some beginnings in providing this link. The relation between the analysis of this paper and these contributions will be noted below.

The approach we propose to follow here is the traditional one of economics, namely, to derive behavior hypotheses and their implications from the assumption that the behavior in question coincides with the optimal response to the given environment. In the present instance, the behavior we are concerned with are decisions relating to the scheduling of production, inventory level, and related variables

[1] Numbers in brackets refer to books and other sources listed on pp. 49, 50.

such as employment and overtime; the environment can be characterized in terms of the cost function and of the time path of sales (actual and anticipated); and the decisions are supposed to be such as to minimize the aggregate of all relevant costs.

We are, of course, fully aware of the possible shortcomings and pitfalls in deriving behavior hypotheses from the assumption of optimal behavior, and of extensive arguments which have been advanced for and against this approach. While we do not intend to engage here in that controversy, we should like to stress that in order to believe, as we do, in the general fruitfulness of that approach it is not necessary to suppose that all firms, at all times, behave in accordance with sophisticated profit maximizing rationality, whether by design or by pressure of circumstances. The very fact that operations research has often achieved substantial payoffs is evidence to the contrary. But even this evidence is not inconsistent with the hypothesis that, as a rule and on the average, behavior is broadly responsive to relevant cost and revenue considerations, especially when the magnitudes involved are large. As a matter of fact, some recent developments in operations research [5] provide some interesting support for this conclusion. These considerations lead us to the view that optimal decision (or normative) analysis can be of considerable usefulness in providing at least approximate predictions of actual behavior.

Needless to say the validity of this view can finally be assessed only by empirical tests.[2] The hypotheses derived below are generally formulated in testable form and we hope they will contribute to furthering the empirical work already in course.[3] Actually, as will soon appear, the decision rules resulting from our analysis are similar to some of the hypotheses that have been submitted to empirical tests. Hence, the fact that these hypotheses seem to fit the data well and lead to estimates of parameters which are in line with the implications of our analysis already lends some support for our approach and for the implications we draw from it. But even if optimal behavior did not provide an adequate approximation to actual behavior, our analysis might not be altogether worthless as it would provide the means for assessing the micro and macro economic implications of behavior aimed at minimizing those costs which in our institutional setup are reckoned as private costs.

The overall organization of this paper is as follows. In section II we undertake to characterize briefly the nature of cost functions relevant to production and inventory decisions. Section III deals with the relation between the cost function and production, employment and inventory decision rules. Our main concern here is to exhibit the nature of the decision rules associated with given "plausible" cost functions. However, we shall also find it instructive to follow the opposite path and examine the cost implications of decision rules which have been hypothesized by earlier contributors, by relying again on the assumption that the postulated rules represent optimal behavior. In section IV we examine certain micro economic implications of the decision rules of section III, focusing on the dynamic response of production, employment, and finished goods inventories

---

[2] The significant contributions of Abramovitz [2] and Stanback [33] should be useful in this connection as well as others [6], [15].

[3] A recent bibliography of contributions in the area of inventory behavior, both at the analytical and at the empirical levels, is provided in the appendix to [10]. This work also provides a useful review of a number of recent empirical studies. (See pt. IV, pp. 1–37.)

to fluctuations in sales, both anticipated and not anticipated. In section V we sketch out an extension of the analysis to the behavior of in-process inventory. In section VI we deal similarly with purchased materials inventory and orders for replenishing this inventory from suppliers outside the firm. These orders perform, with respect to purchased materials, much the same function as production decisions perform with respect to final product, and are the link through which fluctuations of a firm's final sales are transmitted to other firms that engage in the higher stages of production. Finally in section VII we indicate how the normative analysis underlying section III can be used to evaluate the private costs resulting from sales fluctuations and from errors in forecasting sales.

Throughout this paper we are concerned exclusively with the response of the firm to exogenously given sales, or more precisely to orders for the firm's output.[4]

II. The Relevant Costs and the Use of Quadratic and Linear Approximations

We may usefully begin by taking an overall look at the nature of the problem involved in selecting optimal production and inventory decisions. For this purpose let us assume initially that we are concerned with a finite horizon divided into say $N+1$ discrete periods numbered from O to N and that the exogenously given demand (and price) for the product (or products) of the firm in each of these periods is known with certainty. The problem is that of choosing the production schedule say P_t, where $t=0, 1, \ldots, N$ that will maximize profits over the horizon. However, with the demand and hence revenue already exogenously determined, profit maximization is equivalent to cost minimization, and the problem can be reduced to that of finding the production plan that will minimize costs—including in these costs the loss of revenue arising if the production schedule is inadequate to satisfy orders in some periods and sales are thus less than orders.

The aggregate cost over the horizon, say C, can be conceived as the sum of the cost C_t incurred in each of the $N+1$ periods $t=0, 1, \ldots, N$ comprising the horizon, and the best plan is the one that minimizes

$$C=\sum_{t=0}^{N} C_t.^{5}$$

Usually N is considered indefinitely large (but for mathematical reasons not infinite). The precise nature of the costs that might be expected to arise in period t from a given production schedule will be examined more closely in the next section. It is clear however that C_t will include, in addition to any fixed costs independent of the production plan, at least the variable cost of production P_t and the cost of carrying the inventory H_t which is determined by cumulated production scheduled from period O to $t-1$ plus H_0 minus cumulated sales in the same

[4] In the sequel paper, on the other hand, we focus on the macro economic implications of the decision rules derived here and we must take into account the fact that the behavior of orders is itself influenced by that of production.

[5] If capital has a (cash or opportunity) cost, which can be measured by some interest rate r, the best plan will minimize the present value rather than the sum of costs. We shall hereafter neglect this refinement which does not affect the substance of the argument. Actually forgone interest is included as a cost of holding inventory and this has implications similar to discounting.

interval. In other words, C_t will be some function of P_t and of H_t as well as possibly of other variables to be considered later.

In general, the task of choosing the best course of action—such as the best production schedule—is a very exacting one since the actions taken in any given period t will not only affect C_t directly but will also affect the conditions prevailing in later periods and hence the corresponding later costs. Also in determining costs there is a complex interaction with other decision variables at the same time. Finally, for the decision analysis to be of practical value it must be able to handle the case in which future sales are not known with certainty but can, at best, be forecast subject to errors, possibly quite large ones. It should be recognized that at any given point of time a firm needs to be concerned only with what is to be done at that time. (Cf. Modigliani and Cohen [30].) Accordingly a workable solution to the problem requires only a procedure for determining the initial phase of the optimum production schedule, P_t where t is the current time period, and possibly forecasting ahead several periods what production action will be taken where this is necessary for placing purchase orders involving long leadtimes. However, given the interrelation between the decisions at various points of time, it is not obvious that the problem of finding a solution for P_t is in general appreciably simpler than that of finding the complete optimum schedule.

It is only in recent years that systematic attempts have been made at attacking complex problems arising from a sequence of interrelated decisions, and workable solutions have been obtained for certain specific cases—though some of these solutions pose very serious computational problems. (See, e.g. [3], [4], [17], [35], and [36].)

In particular in [1], [14], and [16] we have been able to obtain a complete and relatively simple solution to this problem which is applicable whenever the "criterion function," i.e., the function to be maximized or minimized, can be approximated by a quadratic form meeting certain specifications,[6] and the variables appearing in this function are subject to linear constraints. This solution is directly applicable—and has been applied by operating companies—to the production scheduling problem outlined above where the problem meets the following specifications: (1) The cost C_t incurred in any period can be expressed (at least approximately) as a quadratic function of the controlled variables (e.g., production, inventory, employment) and of the uncontrolled variables (e.g., quantity demanded in each period); (2) certain of the parameters of this quadratic function assume the same value for all points of time within the "practically relevant" horizon (which generally turns out to be short even though the actual horizon is long); (3) any constraints between the variables can be adequately approximated by linear equations.

In attacking the problem of the relation between cost functions and decision rules we will rely on the above mentioned quadratic decision analysis. We will assume that cost functions satisfy at least approximately the above specifications and will accordingly rely on the solution to the problem developed in [14]. This strategic decision is prompted by two major considerations. In the first place, the proposed quadratic cost function is in our view sufficiently flexible to

[6] Perhaps the authors should indicate their awareness of important problems that can *not* be adequately approximated in this fashion. But even this relatively simple decision analysis has yet to be fully exploited.

approximate locally a large class of realistic cost functions. The applications developed in [14] provide some support for this view. In the second place, the mathematical model has a number of very convenient analytical properties. (*a*) It is possible to obtain an explicit solution in the form of an optimal decision rule. (*b*) This decision rule takes the form of a linear equation relating the decision variable, e.g. production in a given period, to lagged values of decision variables and to the uncontrolled variables (lagged, current, and anticipated). (*c*) The coefficients of this linear decision rule can be derived from the cost function with relatively modest computational costs and are the same for all periods, as long as the cost function is unchanged. Hence the task of calculating from the decision rule the optimal decision for the "current" period is a very simple one, and the optimal decision for later periods implied by the optimal solution can be computed, if needed, just as easily by applying the decision rule sequentially. (*d*) The last two properties imply that the decision rules can be cast in the form of linear difference equations with constant coefficients. This type of equation is extremely convenient for the analysis of the dynamic implications of optimal behavior, both at the level of the single firm and for the economic system as a whole. It also greatly facilitates the task of empirical testing and the estimation of parameters. (*e*) The decision rules will lead to optimal decisions even if the future values of the uncontrolled variables (e.g. the demand in each period) is uncertain and must, therefore, be treated as a random variable—provided, in this case, the criterion to be minimized can be taken as the mathematical expectation of costs. The consequence of thus explicitly introducing uncertainty into the problem is that the value of any uncontrolled random variable that appears in the decision rule is now the mathematical expectation of this variable, or more precisely, our best estimate of its expectation.

The analysis developed in [14] has one further implication which we shall exploit in the next section. It can be shown that any decision rule having the general form indicated in (*d*) above (linear difference equation with constant coefficients) can be regarded as the optimal solution for some (not necessarily unique) corresponding quadratic criterion. Furthermore, the nature of the quadratic criterion can generally be inferred from such a decision rule. Hence, as long as we are willing to assume quadratic cost functions, linear constraints and optimal behavior, we can throw some light on the nature of the cost function implicit in any proposed hypothetical decision rule of the above type.

III. Cost Structures and Production, Finished Goods Inventory, and Employment Decision Rules

A number of authors who have been concerned with analyzing the nature of inventory cycles or with understanding production and inventory decisions of firms have in the past hypothesized production decision rules taking the form of linear difference equations—even though usually without explicitly relating these rules to cost considerations. We propose to start out by taking a closer look at some of these hypotheses to see what kinds of (quadratic) cost structures they imply.

One of the first writers who relied explicitly on decision rules of the type under consideration was Metzler [22], [23]. He assumed that production decisions are made at discrete points of time and hypothesized that the production scheduled at the beginning of the period, P_t, would be such as to cover anticipated sales in the period, say $\hat{S}_t$, and to bring terminal inventories H_t to a desired level, say H_t^d. Thus

III–1 $$P_t = \hat{S}_t + (H_t^d - H_{t-1}).$$

Initially, he assumed that the desired level of inventories was proportional to anticipated sales, but later replaced the proportionality assumption with the less stringent one of linearity, i.e.

$$H_t^d = C_1 + C_2\hat{S}_t, \qquad C_2 \geq 0.$$

Substituting from this equation into III–1 we get the Metzler production decision rule exhibited in the first row of table I, under column (4). This rule is clearly linear with constant coefficients and we may therefore inquire as to the nature of the quadratic cost function for which the hypothesized behavior is optimal. The answer to this question is provided in the first row under column (3). We may interpret this cost function as stating that the total cost over the horizon is the sum of the costs incurred in each period $t=0, \ldots, N$, and that the cost incurred in period t has the following components: (1) A fixed cost C_0 independent of production and inventory decisions, (2) a variable production cost proportional to output, represented by C_3P_t, and implying a constant marginal cost C_3; (3) a cost related to the level of inventories, reaching its minimum when inventories are at the level H^d, and rising continuously for deviations from this level in either direction.

We do not propose to enter here into a full fledged discussion of the methods by which the cost structure of column (3) can be inferred from the decision rule of column (4). We can, however, show that the stated cost structure is at least consistent with the decision rule, by establishing that the formula in column (4) yields in fact a production schedule which minimizes the total cost of column (3) and hence is optimal for the given cost function. This task is quite easy for this particular cost structure and will be carried out here in the hope of providing thereby a better understanding of the relation between cost structures and decision rules. For the other cost functions discussed in this paper the decision rule will instead be presented without explicit derivation and the reader is referred to appendix A and to [12], where general methods for deriving decision rules from quadratic criteria are developed.

TABLE I

COST STRUCTURES AND CORRESPONDING DECISION RULES

Model (1)	Authors (2)	Cost Structures (3)	Decision Rules (4)
1	Metzler	$C = \sum_{t=0}^{N}\left[C_5(H_t - C_1 - C_2S_t)^2 + C_0 + C_3P_t\right]$	$P_t = \hat{S}_t - (H_{t-1} - C_1 - C_2\hat{S}_t)$
2	Whitin	$C = \sum_{t=0}^{N}\left[C_5(H_t - C_4\sqrt{S_t})^2 + C_0 + C_3P_t\right]$	$P_t = \hat{S}_t - (H_{t-1} - C_4\sqrt{\hat{S}_t})$
3	Modigliani Sauerlender	$C = \sum_{t=1}^{N}\left[C_5(H_t - C_1 - C_2S_t)^2 + C_0 + C_3P_t + C_6P_t^2\right]$	$P_t = \sum_{i=0}^{N} w_i\hat{S}_{t+i} - C_{12}(H_{t-1} - C_1 - C_2 \sum_{i=0}^{N} y_i\hat{S}_{t+i})$
4	Holt Simon Mills	$C = \sum_{t=0}^{N}\left[C_5(H_t - C_1 - C_2S_t)^2 + C_0 + C_3P_t + C_7(\Delta P_{t-1})^2\right]$	$P_t = \sum_{i=0}^{N} w_i\hat{S}_{t+i} - C_{12}(H_{t-1} - C_1 - C_2 \sum_{i=0}^{N} y_i\hat{S}_{t+i}) + C_{14}P_{t-1}$
5	Holt Modigliani Muth Simon	$C = \sum_{t=0}^{N}\left[C_5(H_t - C_1 - C_2S_t)^2 + C_0 + C_3P_t + C_{11}W_t + C_{23}P_t^2\right.$ $\left. + C_8(P_t - C_9W_t)^2 + C_{10}(\Delta W_{t-1})^2\right]$	$P_t = \sum_{i=0}^{N} u_i\hat{S}_{t+i} - C_{14}(H_{t-1} - C_1 - C_2 \sum_{i=0}^{N} q_i\hat{S}_{t+i}) + C_{16}W_{t-1} + C_{21}$ $W_t = \sum_{i=0}^{N} v_i\hat{S}_{t+i} - C_{17}(H_{t-1} - C_1 - C_2 \sum_{i=0}^{N} z_i\hat{S}_{t+i}) + C_{18}W_{t-1} + C_{22}$
6	"	" " "	Eliminating W_t from above two rules we obtain: $P_t = \sum_{i=0}^{N}\left[w_i\hat{S}_{t+i} + x_i\hat{S}_{(t-1)+i}\right] - C_{12}(H_{t-1} - C_1 - C_2 \sum_{i=0}^{N} y_i\hat{S}_{t+i}) +$ $+ C_{13}P_{t-1} + C_{20}S_{t-1}$

C = Total cost

H_t = Finished goods inventory, end of the period t

S_t = Sales in period t

$\hat{S}_{t+i}$ = Forecast made in period t of S_{t+i}

P_t = Production in period t

$\Delta P_t = P_{t+1} - P_t$

W_t = Work force in period t

C_i, q_i, u_i, v_i, w_i, x_i, y_i, z_i are constants.

The aggregate cost of column (3) is clearly a function of the $2(N+1)$ decision variables P_t and H_t. We wish to minimize C with respect to these variables but subject to the $N+1$ constraints.

III–2 $$H_t = H_{t-1} + P_t - S_t, \qquad t=0, 1, \ldots, N$$

Assuming for the moment that the S_t are known constants, we can use III–2 to eliminate the variables P_t from C obtaining

$$C = \sum_{t=0}^{N} \left[C_5(H_t - C_1 - C_2 S_t)^2 + C_0 + C_3(H_t - H_{t-1} + S_t) \right].$$

Differentiating C with respect to the remaining decision variables H_t and setting the derivatives equal to zero we find

III–3 $$\frac{\partial C}{\partial H_t} = 2C_5(H_t - C_1 - C_2 S_t) + C_3 - C_3 = 0 \qquad t=0, 1, \ldots, N-1$$
$$\frac{\partial C}{\partial H_N} = 2C_5(H_N - C_1 - C_2 S_N) + C_3 = 0$$

which imply, for the cost minimizing inventories,

III–4 $$H_t = C_1 + C_2 S_t, \qquad t=0, 1, \ldots, N-1.$$

Equation III–4 can be regarded as the optimum inventory decision rule implied by the cost function. If sales are uncertain then S_t must be replaced by $\hat{S}_t$ which stands for anticipated sales or more precisely for the mathematical expectation of anticipated sales. Using III–4 to substitute for H_t in III–2 and solving for P_t, we obtain the optimal production decision rule [7]

III–5 $$P_t = S_t + (C_1 + C_2 S_t - H_{t-1})$$

which is precisely the Metzler rule of column (4).

Metzler's analysis did not explicitly relate the desired inventory H^d to cost considerations, and thus provides no explanation for the inventory cost component appearing in the cost function implied by his rule. This task was undertaken at least in part by Whitin in the course of an investigation of certain aspects of optimal inventory management [37]. He concluded that the inventory resulting from production or procurement in *optimal* lot sizes and the *optimum* inventory held as a buffer against the uncertainty of sales should, in certain common circumstances, be proportional to the square root of forecast sales. He further identified the optimal inventory resulting from the above considerations with Metzler's H_t^d [8] and concluded that the decision rule should take the form shown in the second row of column (4). Actually, this conclusion is not fully warranted for the square root relation between (average) inventory and sales is valid only in "steady state", i.e., when sales are anticipated constant and *have been* constant for a sufficiently long time so that every item in the inventory has reached the reorder point at least once. Insofar as the optimal lot for some items lasts appreciably longer than the unit

[7] This equation does not hold for $t=N$, i.e. for the "terminal" period of the horizon. But since in general the horizon is indefinitely large the production (or inventory) rule for the terminal period is of no particular interest.

[8] The relation between H^d and optimal average buffer was also investigated by Mills, [25] and [26].

period t, a change in the level of sales, even if regarded as permanent, would cause inventories to move only gradually toward the new steady state level. In any event, if we take the Whitin rule as given and apply to it the quadratic analysis we find that the implicit cost structure which is minimized by this rule is that exhibited in the second row of column (3).

Since Metzler's and Whitin's rules are quite similar, several observations apply to both. Note that their cost structures imply that there are no economies to be reaped by "smoothing" production, i.e., avoiding swings in the rate of production from period to period. This follows from the fact that the marginal cost of production (namely C_3) is the same in every period, independent of the rate of production in the period. Thus shifting a unit of production from one period to another does not affect total costs. In other words, the total cost of production over the horizon depends only on the total amount produced but not on the way this total is distributed between periods, whether it is spread evenly or heavily bunched in some periods. This lack of incentives to smooth can, of course, be traced back to the postulated production decision rule according to which the rate of production in any given period is controlled exclusively by the sales of that one period and by the requirement of making up *fully* within the period any discrepancy between actual and optimal inventories. On *a priori* grounds one would not expect to find very many business firms with cost functions which imposed no penalties on production fluctuations and, correspondingly, one would expect to find few business firms following the production decision rules in the exact form originally hypothesized by Metzler and Whitin.

The desirability of smoothing production and its implications for the behavior of inventories were stressed by Modigliani and Sauerlander in [28]. Modigliani and Hohn [29] and later several other authors (e.g., Cooper and Charnes [7]) related the economies of production smoothing explicitly to nonlinearities in the cost function, causing the marginal cost of production in each period to increase with the rate of production in the period. Such convex cost functions, reflecting presumably increasing inefficiencies as production is increased relative to capacity, penalize unevenness in the rate of production. Hence the optimum production schedule could be expected to smooth out fluctuations in sales even at the expense of larger and more costly deviations of inventories from the optimum level. If we approximate such a nonlinear cost function by a quadratic expression, we obtain the cost structure exhibited in row (3) of column (3). The corresponding decision rule is shown in column (4).

There are two important differences to note in contrast to the Metzler-Whitin decision rules. First, the production decision depends not on the sales forecast of the next period only, but on a weighted average of the sales forecasts for many future periods. This makes production less responsive to the sales forecast in any one period and hence production is smoother. It can also be shown (see appendix A) that the weights attached to future sales, w_t, decline exponentially so that sales beyond some horizon, which is usually rather short, have a negligible effect on current production and can be altogether disregarded. Second, and even more important, there is a coefficient C_{12} multiplying the discrepancy between the inventory on hand and the optimum inventory, and this coefficient can be shown to be necessarily less than unity. This means that only a fraction of any emerging discrepancy is scheduled to be corrected in the course of each period. Thus the process of inventory correction occurs gradually and smoothly in time, with a speed that depends on the coefficient C_{12}.

Another kind of penalty for fluctuating production, studied by Holt and Simon [13] and by Mills [25], [26] is embodied in the cost function shown in row (4) of column (3). In this model, as in models 1 and 2, production costs rise linearly with the production rate so that there is no penalty on this score from fluctuations in the rate of production. However, an additional cost component is introduced which depends directly on changes in the rate of production, and which is meant to reflect expenses that might be incurred in the process of stepping up or down the rate of production. The corresponding optimal decision rule is shown in column (4). The penalties for fluctuating production incorporated in models 3 and 4 can be introduced simultaneously and in this case the decision rule turns out to have the same form as that of model 4.

Cost structures of type 3 and 4 would appear on a priori grounds to be generally more realistic than those of type 1 or 2. Hence we would expect to find production managers concerned with forecasting sales not just one but several periods ahead and not trying to make a complete inventory adjustment in one period but instead spreading the adjustment over several decision periods. Undoubtedly there are some production operations in which the penalty for production fluctuations are so small that the Metzler-Whitin type of model is applicable, but these would seem to be the exception rather than the rule. However, as will appear later, that type of model is directly applicable to purchasing decisions.

The above models throw considerable light on the dynamic relationships between production and inventory, but some other important variables such as employment level, and amount of overtime and slack time (i.e., time when the work force is being paid but not utilized) are very closely related to the production and inventory decision and it would be desirable to extend the analysis to include these considerations as well. An empirical and theoretical analysis made by Holt, Modigliani, Muth, and Simon [14] has introduced work force as a second decision variable to determine how a manager should make the interacting decisions on production level and work force taking into account his inventory position. The cost function shown as model 5, column (3), is designed to reflect a number of cost considerations which are spelled out in some detail in [14]. Briefly, the inventory cost component incorporated in the first square term, represents the balance of several conflicting forces.[9] The higher the level of inventories the higher the holding costs (e.g., storage, insurance obsolescence, tied-up funds); on the other hand, as the aggregate level of inventories is decreased stock-outs on individual items and delays in filling orders will occur with increasing frequency, generating corresponding costs. Also too low a level of inventories may lead to higher costs by forcing production in uneconomically small lots. The sum of these costs tends to produce a U-shaped curve reaching a minimum for some level of inventories in relation to sales; the minimum cost inventory is here

[9] This model can be applied to production to order, as well as to stock. In this case the variable S_t should be interpreted as orders rather than sales and the variable H_t as the net inventory position, i.e. inventory on hand less unfilled orders. H_t will then be normally negative and a fall in it will imply lengthening of delivery leadtimes.

assumed to be related linearly to sales although it is possible to relax this assumption considerably. The remaining terms of the cost function, excepting the last, reflect overhead costs, material costs, regular payroll costs, and the consequences of increasing production for a given labor force and given capacity which leads to rising marginal costs because of the increasing incidence of overtime and decreased labor productivity beyond some point. The last component reflects costs related to increasing or decreasing the size of the labor force, such as hiring, training, layoff and morale costs.

The optimal decisions for production and for the size of the labor force implied by this cost function are shown in column (4). Both decisions depend on the sales forecasts, the initial inventory level and the initial size of work force, but different weights are attached to these variables in making the two decisions. The appearance of the initial labor force in the production rule reflects the fact that the optimum level of production depends on the work force on hand, which in turn is affected by the initial work force because of the penalties attached to hiring and firing. Since these two decision rules are continually interacting with each other in a complex fashion, it is quite difficult to visualize their dynamic implications. This task can be simplified by eliminating the work force term from the production rule, which can be done with appropriate algebraic manipulations on the assumption that both rules are consistently and strictly adhered to. Under this assumption the production decision rule can be restated in the form shown in the last row of column (4) which turns out to have the same general form as that implied by model 4.

Cost model 5—of which the earlier models can be regarded as special cases—provides a good illustration of the complex and varied type of cost considerations that can be handled by our approach. But the method can in principle handle much more complex cost structures. One limitation of the models considered above that deserves particular mention is the implicit assumption that the time required for the production process does not significantly exceed one decision period. This does not necessarily imply that the production period is short since the decision period itself may be a long one. Presumably a long decision period would be selected if accurate forecasts of orders could be made over a fairly long time span and the cost of decisionmaking and changing plans is relevantly high.

In cases in which the production time p is only a few decision periods, the decision rules can be modified to yield decisions that are very nearly optimal. This is done by sequential application of the rule to determine the desired finished goods production for the period $(t+p-1)$, and this indicates the production to be started in the current period t. However, when the production time extends to many decision periods as is characteristic of many durable goods industries this approach becomes inadequate, because the costs of work force and overtime for a period would not be adequately reflected by a cost function expressed in terms of the finished goods production of a single period.

The question arises as to whether the quadratic cost function and linear dynamic relations are adequate to handle the decision analysis for production times lasting many periods. Although this case has not been developed in detail, enough exploratory work has been done to indicate that an adequate representation can be expressed in this mathematical form.[10] We will briefly sketch one approach. Since industries with long production times tend to have relatively high in-process inventories, this sketch will prove useful later when we consider in-process inventories.

Where the production process requires p decision periods to complete a unit of product and deliver it to finished goods inventory we would expect to find time patterns of labor input and material usage over a sequence of periods (t—p) . . . t as shown, for example, in figure 1. Of course, such a pattern is not rigid but will depend on the production scheduling requirements prevailing at the time. Such time profiles should be interpreted as the mathematical expectations of labor and materials usages at different times in the production process. If for the moment we assume that these patterns are constant, we can write expressions to show the labor requirement L_t and the materials usage U_t as functions of the finished goods production of the following p periods:

III-6 $$L_t=\sum_{i=0}^{p-1} l_i P_{t+i}$$

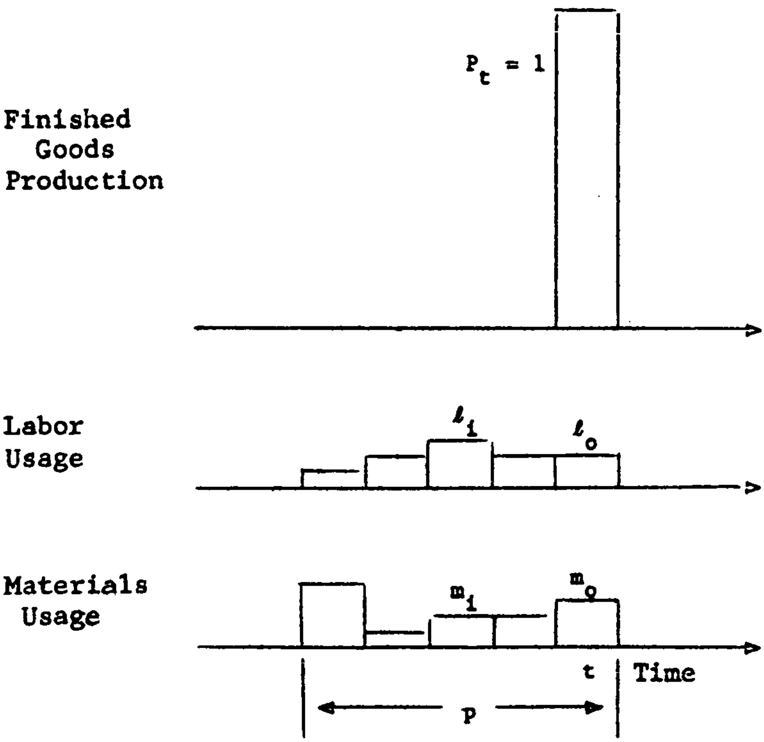
III-7 $$U_t=\sum_{i=0}^{p-1} m_i P_{t+i}$$

where the l_i and m_i are positive constants. Clearly the aggregate labor requirements L_t in a period is more relevant for determining the best work force decision than would be the units of product that happen to be completed in the period. Since these equations are simple linear constraints they can be readily incorporated in the decision analysis and the labor costs expressed in terms of L_t. The analysis of the long leadtime case may well require other modifications of the decision model. We do not propose however to pursue this analysis any further since we feel that the range of cost models explicitly examined in table I is sufficiently broad to give an adequate picture of the type of behavior implied by our approach.

[10] We might note in passing that the sales price might be introduced as another decision variable for the firm by introducing a constant-slope linear demand function whose height was exogenously determined. The revenue function which would be quadratic could be combined with the cost function, and decision rules obtained for maximizing profit.

FIGURE 1

TIME PROFILES OF MATERIALS AND LABOR USAGE FOR A UNIT OF FINISHED GOODS PRODUCTION



It may be of some interest, in this connection, that a number of firms have found the cost function of model 5 adequate to approximate their cost structure and are explicitly employing the corresponding decision rules. These hypotheses seem also to receive some support from a number of empirical studies. To be sure, there have been no explicit tests of these hypotheses or of the underlying approach. As a matter of fact, Mills [26] seems to be the only author who has attempted to test a hypothesis explicitly derived from cost considerations. As noted earlier, his cost structure was of the general type of model 4, and the hypothesis tested was accordingly a variant of the corresponding decision rule (which under certain assumptions was shown to be implied also by model 5). His results, based on a sample of individual firms, are broadly consistent with our hypotheses, except for the prevailingly wrong sign of the lagged production term which might, however, be accounted for by a bias in the estimating procedure.[11]

[11] Because Mills decided to work with first differences, his dependent variable is $P_t - P_{t-1}$, and the lagged production term becomes $P_{t-1} - P_{t-2}$. The regression coefficient for this variable will tend to be downward biased if e.g. there were errors of measurement in the variable P. The presence of bias from this source would help to explain why the regression coefficient became generally positive when shifting from monthly to quarterly data.

Other empirical studies—such as those reported in [8], [9], [18], [24], [32], and [34], to cite only the most recent ones—have typically used inventories or inventory investment rather than production as the dependent variable. In our table we have not exhibited inventory decision rules because, when sales are uncertain, inventory is a random variable reflecting sales fluctuations and only the expected value of inventories, say $\hat{H}_t$, can be regarded as a decision variable. The actual level of inventories, H_t, fluctuates with the random errors in sales expectations. However, the behavior of inventories implied by our analysis can be readily derived from the production rule and the inventory identity III–2. In particular, for decision rule 4 (and 6), we find

III–8 $$\hat{H}_t = C_{12}C_1 + (1 + C_{12}C_2)\sum_{i=0}^{N} w_i \hat{S}_{t+i} - \hat{S}_t + (1 - C_{12})H_{t-1} + C_{14}P_{t-1};$$

and $$H_t = \hat{H}_t + \hat{S}_t - S_t.$$

The hypotheses tested in the empirical studies cited earlier, which have been chosen mostly on general grounds of "plausibility" and ready availability of data, bear only partial resemblance to III–8. The quantity

$$\sum_{i=0}^{N} w_i \hat{S}_{t+i}$$

has been variously replaced (explicitly or implicitly) by past sales, current sales, unfilled orders, new orders, or combinations of these variables. The lagged production term has been dropped or has been replaced by $S_{t-1} + \Delta H_{t-1}$. Also the data used relate to broad aggregates of firms, which in itself poses a problem, unless the coefficients of the decision rules are roughly similar for all firms. Last but not least, the time unit utilized, generally a quarter, need not be an adequate approximation to our decision period.[12] For these reasons there would be little point in attempting a detailed reconciliation between the empirical results and our hypotheses. It seems, however, safe to conclude that the empirical evidence assembled is at least broadly consistent with our models, both in terms of sign and order of magnitude of the coefficients. In particular, all of the studies with which we are familiar have produced estimates of the coefficient C_{12} well below unity, confirming the importance of the gradual inventory adjustment. Indeed in some instances the estimate of this coefficient seems surprisingly low.

Needless to say the various tests to which we have referred can by no means be regarded as conclusive and there is need for further and more stringent tests. As a matter of fact one of the most important uses of the type of analysis developed here should be to further empiri-

[12] It is, of course, possible to carry out tests of our hypothesis even with data relating to time periods different from the "decision period," especially since the decision period is to some extent, an artificial construct. However, it is then necessary to deal explicitly with the problem of "time aggregation" and to modify the hypothesis appropriately. In particular the coefficient C_{12} will be larger the longer the time unit. It should be pointed out in this connection that Metzler, while assuming a complete adjustment of inventories within each "period," proceeded to infer the length of this period from empirical observations about the typical length of inventory cycles, thus weakening some of the objections which we have raised to his decision rule.

cal work by helping in the crucial task of specifying the model to be estimated. To be sure the behavior hypotheses we have derived are not easy to test explicitly as they involve variables which have generally not been measured in the past and whose measurement present serious problems. Nonetheless the task of translating the hypotheses in terms of available observations, as well as the task of measurement or of selecting suitable proxies where direct measurement is unobtainable, should be facilitated by a fuller understanding of the cost considerations that provide the rationale for the hypothesized behavior.

In any event the evidence available so far provides sufficient ground for confidence in the empirical relevance of our behavior hypotheses to justify examination of certain dynamic implications of this behavior.

IV. Response of Production and Employment to Forecast and Unforecast Fluctuations of Sales

If production decisions are controlled by the rules of table I, what will be the response of production to fluctuations in sales, and what difference does it make whether these fluctuations are fully anticipated or whether instead expectations tend to lag behind the experience? In brief we will show that—

(1) The rules tend to cause a cyclical or wavelike response even to disturbances which are not cyclical.

(2) The amplitude of the wave tends to be greater when the change in sales is unanticipated, i.e. the forecast lags, and especially so when the forecast for several periods ahead is dominated by the actual sales experience of the immediate past.

(3) Cyclical fluctuations in sales tend to produce a cyclical response in production, which in spite of production smoothing induced by the cost of fluctuations, may amplify the fluctuations of sales, at least for sales fluctuations having a duration of the order usually associated with inventory cycles.

(4) The amplification of cyclical sales fluctuations is increased by forecast errors, as under (2) above.

(5) The amplification is likely to be decreased by lengthening the duration of the cycle.

To establish these results we shall examine in some detail the time path of production generated by two types of sales fluctuations: first, a sudden increase in sales from one constant level to another, and second, cyclical (or more precisely sinusoidal) fluctuations in sales. We will confine our attention to three of the decision rules: first, third, and fifth.[13]

In order to make numerical calculations specific values have been assumed for the coefficients in the cost functions, and these uniquely determine the parameters of the decision rules. Although these cost estimates are based largely on an empirical cost study and hence are not unrealistic, they are intended here to support only general qualitative conclusions. Different cost coefficients would, of course, give quantitatively different decision rules but they would have the same general form.

---

[13] An analysis analogous to that undertaken here, but applied to other decision rules is given in [26] and [31].

RESPONSE TO A SUDDEN INCREASE IN THE LEVEL OF SALES

Although the Metzler rule is not very plausible as a predictor of factory production its simplicity makes it a good place to start. The relation between the time path of sales and of production implied by a given rule can be seen most clearly if in the rule we eliminate the initial inventory term H_{t-1} expressing it in terms of previous sales and sales expectations. For the Metzler rule this elimination is easily accomplished. We first note that in this case equation III–4 implies

$$\hat{H}_t = C_1 + C_2\hat{S}_t \quad \text{and} \quad H_t = C_1 + C_2\hat{S}_t + (\hat{S}_t - S_{t-1}).$$

Since this last equation holds for all t and in particular for t–1, we can use it to eliminate H_{t-1} from the production rule III–5, obtaining

IV–1 $$P_t = \hat{S}_t + (S_{t-1} - \hat{S}_{t-1}) + C_2(\hat{S}_t - \hat{S}_{t-1}).$$

In this form the following interpretation can be given to the rule: Production is equal to forecast sales for the period, plus the previous forecast error, plus an acceleration component equal to the change in anticipated sales multiplied by the desired (marginal) inventory-sales ratio, C_2.

The dynamic response of this decision rule to a sudden increase in sales is shown for three different forecasts in figure 2.[14] If the firm could forecast sales for the current period perfectly, i.e. $\hat{S}_t = S_t$, the first parentheses in IV–1 would disappear.

IV–2 $$P_t = S_t + C_2(S_t - S_{t-1}).$$

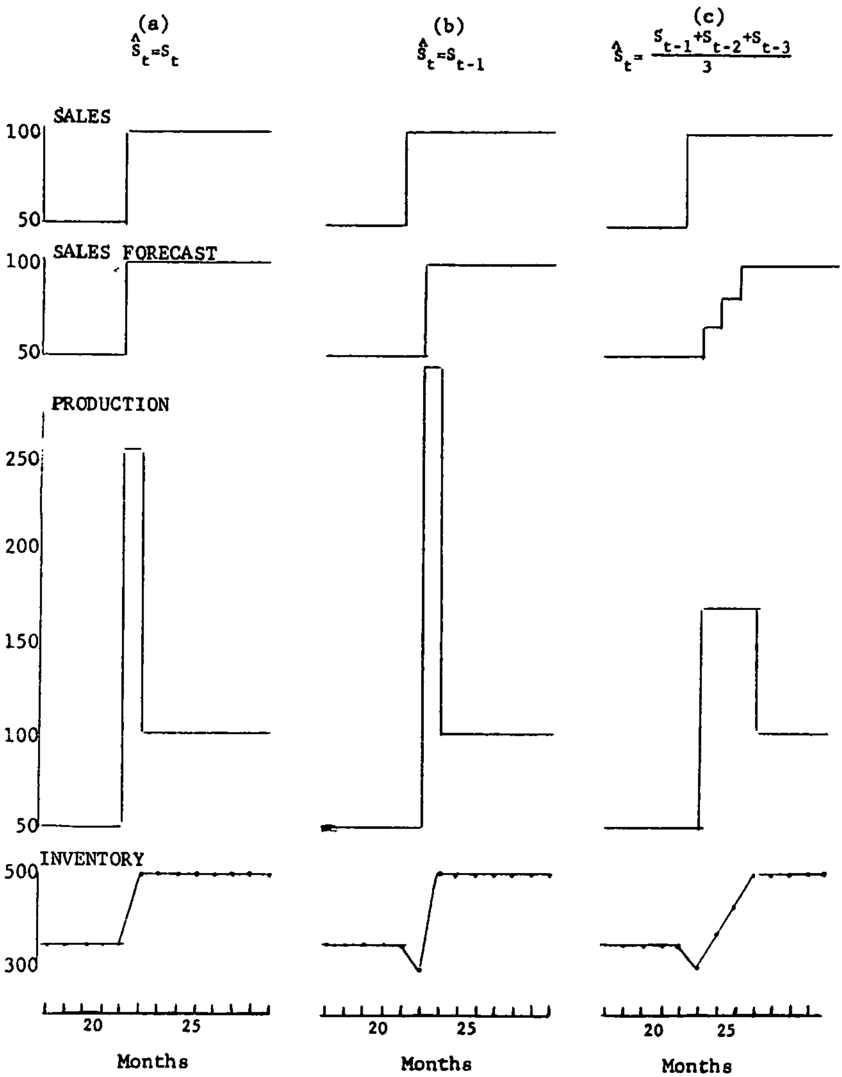
As shown in figure 2a, for this case the response of production to a step in sales is a half wave; production rises at first above the new level of sales to accumulate the additional inventory $C_2\Delta S_t$ needed to service the increased level of sales, then relapses to the level S_t. Furthermore, because the whole adjustment is scheduled in a *single* period—the period of the correctly forecast change in sales—the wave is extremely sharp and depends entirely on the acceleration coefficients C_2, which in our example is taken as 3 (i.e. 3 months' sales).

---

[14] In the following examples of dynamic response, sets of reasonable numerical values have been given the parameters assuming that the decision period is monthly. Sales, forecasts of sales, and production are in units of product per month; inventory is in units of (finished) product; and work force is in hundreds of men. The cost functions and decision rules with numerical values are given in appendix B.

FIGURE 2

RESPONSE OF DECISION RULE 1
TO A SUDDEN INCREASE IN SALES



Perfect forecasts are, of course, unlikely but not a great deal is known about the dynamics involved in the formation of expectations. However, we know that many business firms use forecasting methods that rely heavily on moving averages of recent past sales. Consider first the case in which the sales of the previous period are used as a forecast of sales for the following period, i.e., $\hat{S}_t = S_{t-1}$. Substituting this forecast relationship in IV–1 yields

$$\text{IV–3} \qquad P_t = S_{t-1} + (1 + C_2)(S_{t-1} - S_{t-2}).$$

The pattern of response implied by this equation is shown in figure 2b, and is similar to that of figure 2a, except that (1) it lags behind by one period, and (2) the half wave rises further above the new sales level—this time by $(1+C_2)\Delta S_{t-1}$. This is because the acceleration effect is reinforced by the need to correct the unintended loss of inventories in period t, which correction is also scheduled within the single period t+1.

Since sales are subject to erratic fluctuations, the forecasting method just considered would tend to yield excessively volatile forecasts by being overly responsive to the most recent sales experience. Accordingly, sales forecasts are frequently based on a moving average of sales over several previous periods. Such forecasting is indicated in IV–4, for an I period moving average.

$$\text{IV–4} \qquad \hat{S}_t = 1/I \sum_{i=1}^{I} S_{t-i}.$$

When such forecasts are used with the decision rule of equation IV–1, we obtain equation IV–5.

$$\text{IV–5} \qquad P_t = S_{t-1} + (1 + C_2)(1/I)(S_{t-1} - S_{t-1-I}).$$

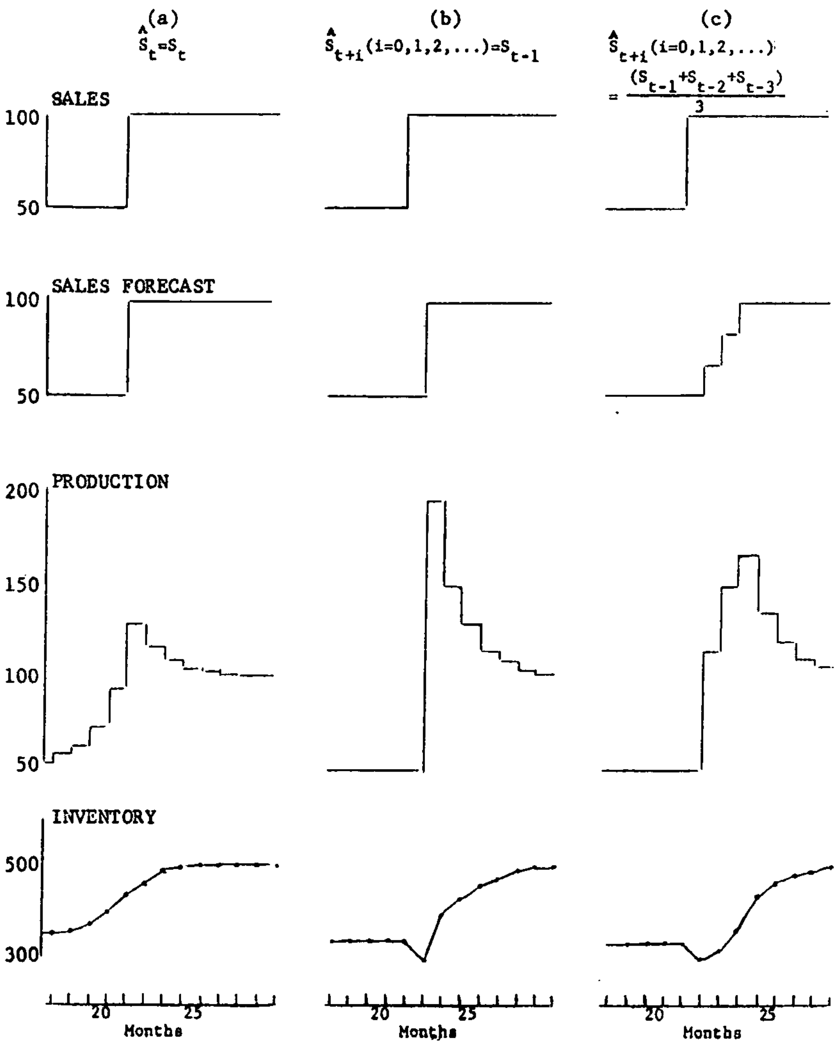
Figure 2c shows the response when the forecast is based on a three period moving average, i.e. I=3. Again inventory is drawn down initially since the sudden increase in sales is not anticipated, but the forecast of sales does not immediately adjust to the new level taking three periods to do so. As the forecast errors gradually decline, production to make up for these errors declines but this happens to be exactly offset by the tendency of the rising forecasts to increase production. The result is still a half wave in production but it is now spread over three periods (generally over I periods), so that the height of the wave is only one-third as high as in case b, where $\hat{S}_t = S_{t-1}$.

Thus, for the Metzler model we can draw the following conclusions. The inventory accelerator causes production to increase temporarily more than the causal increase in sales so that the characteristic response is an oscillatory overshoot, even when the rise is perfectly forecast. When the rise is not forecast, the overshoot is even greater because of the need to make up the unintended depletion of inventories. When the forecast is based on past history, a short moving average forecast is highly responsive and yields fast production changes which are correspondingly violent. Forecasts based on more periods of past history lead to a production response of longer duration, but correspondingly less violent.

The absence of any cost penalty for fluctuations in production makes decision rule 1 of limited significance for the prediction of actual production response. In contrast, the third decision rule takes into account the cost of fluctuations and we would anticipate a less violent dynamic response for this case. If we make the same assumptions about forecasts and substitute them in decision rule 3, we obtain the dynamic responses shown in figure 3. In order to make the examples as comparable as possible, we have assumed that the coefficient C_2 relating the desired level of inventory to sales rate is the same in all cases. The perfect forecast case, shown in figure3(a), illustrates clearly

FIGURE 3

RESPONSE OF DECISION RULE 3
TO A SUDDEN INCREASE IN SALES



the production smoothing which is the result of cost penalties on production fluctuations. The adjustment of the inventory level is a gradual one instead of a precipitate one as was found in model 1. Such deviations of the inventory from the desired level are tolerated in the interests of avoiding costly fluctuations in the production level. Correspondingly, the production response is spread over many periods. Also since sales are forecast perfectly the production response actually starts long before the increase in sales.

Figure 3(b) shows the results of using the previous period sales as a forecast of sales in the next period as well as in all of the relevant future periods. Such forecasts, of course, are highly volatile and contribute to making the production response much more violent, as shown in figure 3(b). When the forecasts of sales in future periods are based on 3 months moving averages of past sales, the production response shown in figure 3(c) is much less violent, though its amplitude remains greater than with a perfect forecast.

Next we consider decision rule 5 in which both production and work force decisions are made in response to increased sales. Implicit in these two decisions is the amount of overtime work which determines the length of the workweek. Again we have chosen cost coefficients similar to those used in previous rules, in order to make results as comparable as possible. Figure 4(a) shows the response when the sales are perfectly forecast. In spite of the fact that the sales increase was perfectly anticipated, the familiar overshoot in production occurs in order to build inventory to the desired higher level. The work force response, shown in the bottom panel is much smoother, and, for the cost structure used in our illustration, does not overshoot but gradually builds up to the new level. The discrepancy between the employment and production patterns is accounted for by reliance on overtime, whose behavior is not shown explicitly in our graph.

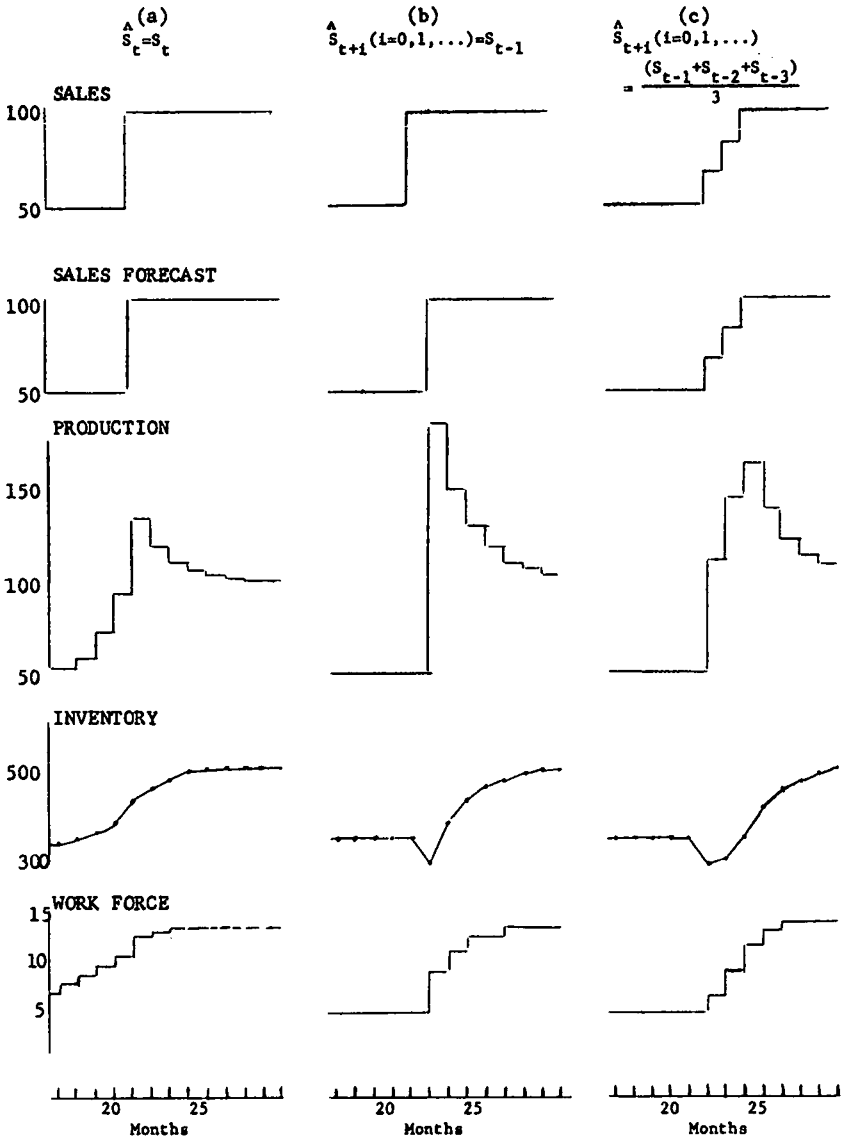
Although with the assumed cost function the response of production is a half wave even for model 5, this need not be the case. For certain combinations of cost parameters the response for this model might instead be a wavelike or cyclical motion with production repeatedly overshooting and undershooting the new level of sales. This kind of behavior can also occur under decision rule 4. However, the cycle will normally be heavily damped and die out in a few cycles.

The effect of a sales forecast lagging sales, shown in panels (b) and (c), is quite similar to that observed for model 3. The response lags and its amplitude of fluctuations is increased, the more so the shorter the moving average on which the forecast is based.

One characteristic common to all of the models deserves emphasis. The very real cost considerations that lead business firms to desire to hold higher inventories at higher sales rates has the consequence of inducing initially a larger change in production than the initial sales disturbance. Because of this overshoot even a unidirectional change in sales produces oscillatory response in production which may take either the form of a half wave or of a damped cycle.

FIGURE 4

RESPONSE OF DECISION RULE 5
TO A SUDDEN INCREASE IN SALES



RESPONSE TO SINUSOIDAL FLUCTUATIONS OF SALES

To round out our analysis of the micro dynamic implications of our cost and decision analysis it is interesting to examine the response of production, employment and inventories to the kind of wavelike fluctuations of sales that arise in the course of a business cycle. For analytical purposes this kind of fluctuation can be conveniently represented by a simple sinusoid, i.e., an equation of the form

IV-6 $$S_t = S \sin (2\pi/T)t + \text{constant}$$

where S measures the amplitude of sales fluctuations (i.e., half of the difference between the peak and the trough of sales) and T measures the length of the cycle expressed in months ($2\pi/T$ is the angular velocity of frequency measured in cycles per month).

Since our rules take the form of linear difference equations in which sales act as a forcing function, each of the decision variables (in steady state) will itself trace a sinusoidal cycle of equal length though generally of different amplitude and not necessarily in phase with the cycle of sales. In other words the response of any decision variable such as production will be given by an equation of the form

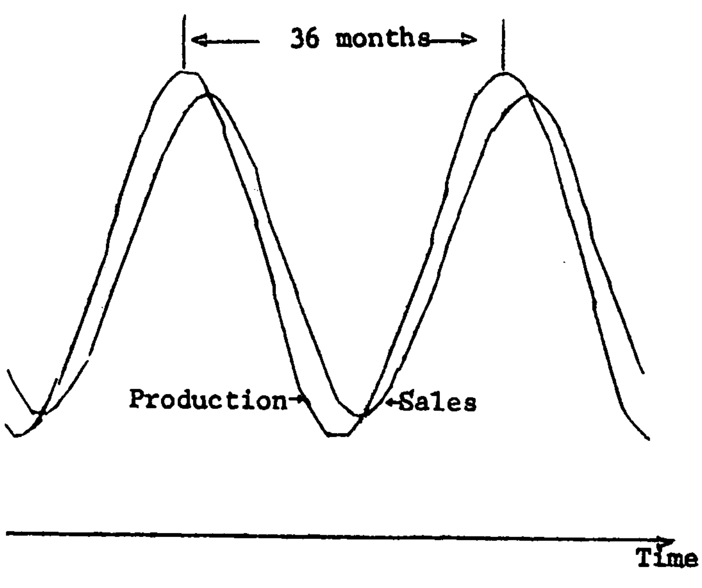
IV-7 $$P_t = P \sin (2\pi/T)(t + L_P) + \text{constant}$$

where P is the amplitude of the response and L_P the lead (if positive or lag if negative) of production relative to the cycle of sales. Particular interest attaches to the ratio P/S which is a measure of "amplification." If this ratio is larger than one, this means that the response amplifies the fluctuation of sales if smaller than one, that it dampens these fluctuations.

Figure 5 provides a graphical illustration of the typical relation between the cycle of sales and the cycle traced by the responding

FIGURE 5

RESPONSE OF DECISION RULE 1 TO PERFECTLY FORECAST SINUSOIDAL SALES



decision variables. The cycle of sales in this illustration has a length of 36 months or 3 years, which is fairly typical of inventory cycles. The graph shows in addition to the behavior of sales, the response of production implied by the simple Metzler rule, when sales are forecast perfectly. In this case the amplification ratio for production, P/S, turns out to be 117 percent and for inventories, H/S, 300 percent.[15] The latter result is easily understandable, for, with a perfect forecast the inventory in the Metzler model is always at the optimum or desired level; namely, C_2S_t, and it will be recalled that in our illustrations C_2 has been assigned the value 3.

These same considerations also explain why the cycle of inventories is precisely in phase with that of sales. On the other hand, production leads sales, and for this particular frequency the lead is approximately 2.6 months. This seemingly paradoxical result can be understood when we recall that under the Metzler rule with perfect forecasts the response of sales is given by IV–2. We see that production is here the sum of two components; namely, sales itself, and the accelerator component which is proportional to the first difference or rate of change of sales. Now it is well known that for sinusoidal functions the rate of change leads the series itself and by about one quarter of a cycle. Thus production being a weighted average of a coincident and a leading series, has itself some lead over sales. It is important to note that this lead is *not* simply the result of perfect forecasts but also will occur with lagging forecasts. This result can also be established more rigorously by substituting IV–6 into the Metzler rule IV–2 obtaining

IV–8

$$P_t = S \sin (2\pi/T)t + C_2S[\sin (2\pi/T)t - \sin (2\pi/T)(t-1)] + \text{constant}$$

Making use of well known trigonometric identities this equation can be rewritten in the form IV–7 where

IV–9

$$P = S[1 + 2C_2(1 + C_2)(1 - \cos 2\pi/T)]^{\frac{1}{2}}$$

and

$$L_P = \frac{T}{2\pi} \arctan \frac{C_2 \sin (2\pi/T)}{1 + C_2[1 - \cos (2\pi/T)]}$$

It is seen from this equation that the lead L_P will depend on the length of the cycle T, and that it will be positive for values of T in the relevant range, say between 6 and 60 months. It is equally apparent that the amplification coefficient P/S will also depend on the length of the cycle T, and will in fact be greater the shorter the cycle, although it will always remain larger than one in the relevant range of T. These results are exhibited in numerical form in the first part of table II in which we have tabulated the amplification coefficient and the lead for both production and inventories implied by the Metzler rule with perfect forecast for selected cycle lengths between 6 and 60 months. It will be seen that the amplification rises rapidly as T shortens, reaching 60 percent for a cycle in the order of 1½ years. This relation between P/S and T is, of course, due to the fact that the

[15] In interpreting this ratio remember that inventory is a stock and sales is a flow.

shorter the cycle the greater is the rate of change of sales per period and hence, also the greater the component of production due to the acceleration effect.

In table II we have also tabulated the same information for rule 3. As one might have expected the presence of cost penalties for production fluctuations leads to a smaller amplification for all values of T covered. What is somewhat surprising, however, is that, in spite of the cost incentive to stabilize production, the amplification coefficient is still larger than one throughout the entire range of cycle lengths covered. In the most relevant range, 2 to 3 years, it is still around 110 percent as compared with 117 to 135 percent for the Metzler rule. It will also be noted that the amplification reaches its maximum for a cycle of about a year. This is somewhat disturbing for 12 months is the usual period of seasonal fluctuations;[16] of course, the result is partly due to the specific choice of cost coefficients. For shorter cycles production tends to dampen the fluctuation of sales as it pays to absorb these fluctuations by allowing inventory to fluctuate. It will also be noted that production again consistently leads sales and in fact by exactly the same lead as under the Metzler rule.

TABLE II.—*Response to perfectly forecast sinusoidal sales*

Cycle length (months)	Production fluctuation (relative to sales)	Production lead (relative to sales in months)	Inventory fluctuation (relative to sales)	Inventory lead (relative to sales in months)	Work force fluctuation (relative to sales divided by labor productivity)	Work force lead (relative to sales in months)
MODEL 1						
60	1.064	2.853	3.000	0.000		
48	1.098	2.787	3.000	.000		
36	1.168	2.646	3.000	.000		
24	1.348	2.345	3.000	.000		
18	1.564	2.048	3.000	.000		
12	2.110	1.564	3.000	.000		
6	3.630	.769	3.000	.000		
MODEL 3						
60	1.036	2.853	2.920	0.811		
48	1.054	2.787	2.877	.811		
36	1.088	2.646	2.788	.807		
24	1.157	2.345	2.561	.810		
18	1.210	2.048	2.299	.810		
12	1.245	1.564	1.780	.812		
6	1.053	.769	.805	.827		
MODEL 5						
60	1.061	2.853	2.991	0.076	0.957	2.853
48	1.091	2.787	2.979	.114	.928	2.787
36	1.148	2.646	2.943	.189	.877	2.646
24	1.264	2.345	2.791	.328	.748	2.345
18	1.349	2.048	2.548	.441	.612	2.048
12	1.394	1.564	1.969	.590	.378	1.564
6	1.125	.769	.840	.753	.102	.769

[16] The period may, of course, be less than 12 months if there is more than one peak in sales in the course of the year.

The last part of table II shows that very similar results are obtained for model 5. However, the amplification is somewhat larger, especially in the most relevant range of cycle lengths. On the other hand, the work force, which is now included as a decision variable, responds with an amplitude [17] which decreases continuously as the length of the cycle shortens, and in relative terms this amplitude is always smaller than that of sales. The difference between the amplitude of production and work force is made up by fluctuations in overtime, which is not shown explicitly. Thus we see that cost minimization calls for the following dynamic responses. For long cycles the work force absorbs the sales fluctuations, and for very short cycles changes in the inventory level absorbs them. For cycles of intermediate length, fluctuations in overtime and slack time reach their peak, and the sales fluctuation is absorbed by a combination of inventory, work force, and overtime-slack time fluctuations.

In order to assess the effect of erroneous and systematically lagging sales forecasts, we show in table III a tabulation analogous to that of table II, but based on the assumption that the sales forecast is a 3-month moving average of past sales. Much as in the case of a one-time change in sales, the principal effect is an increase in the coefficient of amplification for both production and inventories, and a very appreciable one at that. In the case of rule 3 for instance, for a 2-year cycle the amplification in the production response rises from 116 percent with perfect forecast to 179 percent with a moving average forecast, and even for a 3-year cycle we observe an increase from 109 to 145 percent. Very similar results hold for model 5, where we also observe a considerable increase in the amplitude of the employment response. The other effect of a forecast lagging behind sales is to reduce the lead of production on sales.[18] Interestingly enough, even in this case production does not actually lag sales until the cycle gets quite short, less than 2 years; however, for cycles in the critical range of 2 to 3 years the lead is rather negligible—in the order of 1 month.

---

[17] The work force amplitude is divided by the work force amplitude which would have occurred if the sales fluctuation had been absorbed completely by work force fluctuations without overtime or slack time; i.e., the sales amplitude divided by the coefficient of labor productivity.

[18] That the caused variable (production) could lead the causing variable (sales) may appear to pose a dilemma since we are accustomed to associating a time lead with causality. Indeed, this causal interpretation was found to be fully applicable in the earlier case which was considered of a sudden change in sales. But when sinusoidal fluctuations are involved a dynamic system that responds to the rate of change of the causal variable may very well generate variables that lead it.

TABLE III.—*Response to sinusoidal sales forecast by 3-month average of past sales*

MODEL 1

Cycle length (months)	Production fluctuation (relative to sales)	Production lead (relative to sales in months)	Inventory fluctuation (relative to sales)	Inventory lead (relative to sales in months)	Work force fluctuation (relative to sales divided by labor productivity)	Work force lead (relative to sales in months)	Forecast fluctuation (relative to sales)	Forecast lead (relative to sales in months)
60	1.142	2.524	3.014	−2.659			0.996	−2.000
48	1.214	2.314	3.022	−2.655			.994	−2.000
36	1.354	1.949	3.039	−2.646			.990	−2.000
24	1.680	1.276	3.084	−2.622			.977	−2.000
18	2.028	.736	3.140	−2.591			.960	−2.000
12	2.687	−.009	3.260	−2.513			.911	−2.000
6	3.666	−1.000	3.283	−2.250			.667	−2.000
MODEL 3								
60	1.190	2.215	3.020	−4.511			0.996	−2.000
48	1.280	1.899	3.031	−4.485			.994	−2.000
36	1.446	1.388	3.052	−4.432			.990	−2.000
24	1.787	.539	3.098	−4.294			.977	−2.000
18	2.089	−.068	3.138	−4.130			.960	−2.000
12	2.497	−.799	3.157	−3.780			.911	−2.000
6	2.390	−1.530	2.625	−2.920			.667	−2.000
MODEL 5								
60	1.195	2.178	3.019	−4.729	1.007	−2.474	0.996	−2.000
48	1.287	1.850	3.033	−4.707	1.007	−2.513	.994	−2.000
36	1.456	1.318	3.060	−4.651	1.007	−2.567	.990	−2.000
24	1.798	.443	3.114	−4.570	.991	−2.663	.977	−2.000
18	2.090	−.172	3.150	−4.328	.952	−2.713	.960	−2.000
12	2.451	−.894	3.135	−3.932	.815	−2.706	.911	−2.000
6	2.243	−1.575	2.527	−2.962	.413	−2.430	.667	−2.000

For rule 5 we present in table IV one more tabulation, showing the effect of a sales forecast based on a 12-month moving average. This tabulation is of some interest because of the frequent use of 12-month moving averages in an effort to eliminate the effect of seasonal influences from the recent sales experience. The results in this case are by no means clear cut: the amplitude of the production response is increased significantly for a 3-year cycle but somewhat reduced for lengths of 2 years or less. On the other hand, the amplitude of employment is somewhat decreased for the entire range covered. Also, as one might expect, the lead of production over sales is reduced further and actually turns into a lag already for a 3-year period.

TABLE IV.—*Response to sinusoidal sales forecast by a 12-month moving average of past sales*

MODEL 5

Cycle length (months)	Production fluctuation (relative to sales)	Production lead (relative to sales in months)	Inventory fluctuation (relative to sales)	Inventory lead (relative to sales in months)	Work force fluctuation (relative to sales divided by labor productivity)	Work force lead (relative to sales in months)	Forecast fluctuation (relative to sales)	Forecast lead (relative to sales in months)
60	1.341	0.782	3.385	−11.48	0.987	−5.14	0.936	−6.50
48	1.463	.115	3.541	−11.17	.970	−5.23	.901	−6.50
36	1.629	−.832	3.763	−10.60	.925	−5.37	.828	−6.50
24	1.728	−2.070	3.877	−9.58	.748	−5.53	.638	−6.50
18	1.487	−2.650	3.438	−8.75	.504	−5.42	.416	−6.50
12	.800	−1.91	1.706	−6.83	.187	−3.36	.000	−6.50
6	.539	−1.58	1.168	−3.37	.074	−2.30	.000	−6.50

In models 3, 4, and 5 where production smoothing offsets at least partially the tendency of the inventory accelerator to amplify fluctuations, the overall responses depend on the relative cost importance of the two kinds of considerations. In order to show how the optimal dynamic responses would be affected if the cost structures were different from those previously assumed, we show in table V the production response of model 3 for different values of the marginal inventory-sales ratio C_2. This coefficient relates the variable component of desired inventory (which is a stock) to the monthly sales rate (which is a flow). This variable component of inventory is given values ranging from 0.5 to 5.0 months' sales.

TABLE V.—*Influence of the marginal inventory-sales ratio, C_2, on the production response to sinusoidal sales*

MODEL 3 WITH PERFECT FORECASTS

Cycle length (months)	Production fluctuations relative to sales					
	C_2=.5	C_2=1	C_2=2	C_2=3	C_2=4	C_2=5
60	0.978	0.985	1.006	1.036	1.076	1.123
48	.966	.976	1.008	1.054	1.112	1.181
36	.942	.959	1.013	1.088	1.181	1.288
24	.880	.915	1.019	1.157	1.319	1.497
18	.808	.862	1.016	1.210	1.429	1.663
12	.664	.751	.979	1.245	1.529	1.823
6	.386	.506	.773	1.053	1.338	1.626

MODEL 3 WITH 3-MONTH MOVING AVERAGE FORECASTS

Cycle length (months)	C_2=.5	C_2=1	C_2=2	C_2=3	C_2=4	C_2=5
60	1.064	1.085	1.134	1.190	1.252	1.319
48	1.096	1.128	1.200	1.280	1.368	1.461
36	1.159	1.210	1.323	1.446	1.577	1.714
24	1.299	1.390	1.583	1.787	1.997	2.213
18	1.432	1.558	1.819	2.089	2.365	2.644
12	1.615	1.788	2.140	2.497	2.856	3.217
6	1.490	1.670	2.030	2.390	2.751	3.111

It is clear from the top of table V that even when forecast errors are excluded the amplification of production fluctuations can be considerable if the inventory accelerator coefficient C_2 is large. On the other hand if C_2 is low, production fluctuates with a smaller amplitude than sales; i.e., the production smoothing dominates and the fluctuations are attenuated.

The introduction of forecast errors by using a 3-month moving average forecast increases the amplification as shown in the bottom of table V. When C_2 has high values the amplification may become very high indeed in spite of production smoothing, but when C_2 is low the amplification is *relatively* small.

Clearly to draw quantitative conclusions about the production amplification or attenuation of sales fluctuations in various industries, detailed studies of cost structures, decision rules, forecast dynamics, and sales fluctuations would be very useful. Substantially different results can be anticipated for different industries.

Needless to say, our numerical results must be interpreted with caution since they are after all based on specific though hopefully representative sets of cost coefficients.[19] Furthermore the cycle of demand does not possess the degree of regularity we have been assuming, and finally some of the characteristics studied will be influenced when we take into account the effect of production on sales via the income mechanism and the incentive of firms to spend on fixed capital. Nonetheless our tabulations do help in providing at least some qualitative notion about the micro implications of optimal decision rules. Of particular interest in this connection is the production response for it is closely related to the generation of income. Our main result here is that even when there are significant private costs of production fluctuations and hence conscious efforts at stabilizing the rate of production, production and income payments may end up fluctuating more widely than sales. The explanation is to be found in the economic incentive to adjust the level of finished goods inventories so that they will be adequate to service customers, etc., and the resulting acceleration effect which may more than offset the effect of production and employment smoothing. The amplification is further increased to the extent that firms are unable to forecast changes in sales sufficiently in advance to schedule more orderly adjustments. Also to the extent that expectations are overly responsive to the most recent sales experience, the production fluctuations tend to be amplified.

One interesting tentative conclusion suggested by our analysis and tables relates to the likely effect of measures which would change the length of the cycle. If the typical length of the inventory cycle is not less than 2 years, as historical experience seems to suggest, then it would appear that a lengthening of the cycle would for individual firms tend to reduce the amplitude of production fluctuations relative to that of sales.

Up to this point we have concentrated in this paper on the response of production, work force, and finished goods inventory of the firm to sales fluctuations. However, one of the responses that a business firm makes to fluctuations in its sales is to place orders with its sup-

[19] The cost estimates were based on a set of factory accounting data except for the marginal inventory sales ratio C_2. The value of 3 (months' sales) assigned to this coefficient may be representative of the ratio of total inventories to sales but may be too high as an estimate of the prevailing marginal ratio of finished goods inventories to sales.

pliers and these in turn constitute a fluctuating sales load for other business firms. In considering the generation of such orders we need to examine two additional types of inventory held by a firm, in-process inventory and the purchased materials inventory.

V. Fluctuations of In-Process Inventory

The operations research literature deals much more with finished goods inventory at the factory and in warehouse distribution systems than it does with in-process inventory. Of course, many of the decision problems are very similar and then the same mathematical models can be applied, but to a much greater degree the control of in-process inventories involves the whole factory production process which raises complicated systems problems on which a great deal of work remains to be done before optimal solutions are found. However, some important aspects of the determinants of in-process inventories are clear.

Some fundamental relationships can be readily shown by considering the case mentioned at the end of section III in which the production process has a duration of many decision periods but our results will not be limited to this case in applicability.

The investment in in-process inventory is equal to the expenditure on labor L_t plus the utilization of purchased materials U_t minus the finished goods completed within the period P_t, i.e.,

V–1 $$I_t - I_{t-1} = L_t + U_t - P_t$$

where I_t is the in-process inventory [20] at the end of the period t and all variables are expressed in consistent units. This relation derives from the fact that in-process inventory is a stock whose level changes when there is a difference between the inflow to and the outflow from the stock.

Assuming that the production time as reflected in the time profiles of figure 1 is constant, we may substitute expressions for L_t and U_t from III–6 and III–7 to obtain a relationship between inventory investment and finished goods production [21]

V–2 $$I_t - I_{t-1} = \sum_{i=0}^{p-1} (l_i + m_i) P_{t+i} - P_t \, .$$

Although holding the l_i and m_i constant suppresses certain aspects of inventory investment which will be considered later, this assumption will enable us to see clearly the investment that is associated with a process of fixed duration. We would expect no in-process inventory investment when the system is in equilibrium with a constant production level. Hence, as shown in equation V–1, labor input plus materials utilization must equal finished goods production and then from V–2 it follows that there is a restriction on the l_i and m_i coefficients,

V–3 $$\sum_{i=0}^{p-1} (l_i + m_i) = 1.$$

[20] Note that we have explicitly ignored allocated overhead which is usually taken into account by accountants in estimating in-process inventory. Usually overhead is allocated on the basis of L_t anyway.

[21] There are many and complex considerations involved in optimizing the l_i and m_i profiles. We will not consider them here.

Similarly we can write an expression for the in-process inventory level,

V–4 $$I_t=\sum_{i=1}^{p-1}\left[\sum_{j=i}^{p-1}(l_j+m_j)\right]P_{t+i},$$

which is a weighted average of finished goods production in $(p-1)$ future periods where the weights are indicated by the expression within the brackets. This can be written alternatively as

V–5 $$I_t=\sum_{j=1}^{p-1}\left[(l_j+m_j)\sum_{i=1}^{j}P_{t+i}\right].$$

It is easier to see the implications of these relations if we consider the simple case in which the profiles shown in figure 1 are rectangular. Then we obtain for inventory investment

V–6 $$I_t-I_{t-1}=\frac{1}{p}\sum_{i=0}^{p-1}P_{t+i}-P_t$$

which shows that investment in in-process inventory is equal to a kind of distributed rate of change of the production of finished product. The source of this investment is very simply the time delay p that occurs between the initiation of production and its final completion. If we visualize a factory producing at a constant rate and suddenly increasing its level of activity by starting new units in production at a higher rate, then for $p-1$ periods material and labor would be consumed in the production process at a higher rate but there would be no corresponding increase in output. Starting with the next period the output flow rises to the new rate and inventory investment ceases.

The corresponding expression for the inventory level is

V–7 $$I_t=\frac{1}{p}\sum_{i=1}^{p-1}(p-i)P_{t+i}$$

which indicates as before that the inventory is a weighted average of the finished goods production in different future periods with the greatest weight going to the products that are furthest advanced in the production process.

In the steady state the inventory is proportional to the production rate and the inventory-sales ratio rises linearly with the duration of production p

V–8 $$I=\left[\frac{1}{p}\sum_{i=0}^{p=1}(p-i)-1\right]P=\left(\frac{p-1}{2}\right)P.$$

Since we have assumed that labor and materials are added at a uniform rate during the production process, the average time that the in-process inventory for a unit is held is roughly half the production period (if time were continuous, it would be exactly one-half).

In general, the steady state in-process inventory is the product of the average production time $\bar{p}$ and the production rate, specifically from equation V–5

V–9 $$I=\bar{p}P$$

where

V-10 $$\bar{p}=\sum_{j=0}^{p-1}(l_j+m_j)(j).$$

Thus p is a weighted average of the times that various injections of labor and material remain in the production process and the weights, whose sum is unity, are the injections that occur at the various periods of the process.

The significance of the foregoing analysis is: (1) That any lag between a change in labor and material usage and an equal change in finished goods production will result in in-process inventory investment or disinvestment (see equation V-1); and (2) that the steady state in-process inventory depends on the average production time and the finished goods production rate (see equation V-9 and V-10).

Although this analysis drew on an earlier one in which the assumption was made that the production time exceeded the decision period, the results are not limited to this case. Presumably for production planning and control the decision period is broken down into subperiods and the in-process inventory control analysis above can be interpreted as applying to them. For example, the average production time $\bar{p}$ might be less than the decision period as was generally assumed by the models in section III.

With this background we would like to consider the implications of production fluctuations for changes in in-process inventories under several different types of factory operation. It is convenient to classify the methods for changing the production rate of a factory into three pure types recognizing that in actual operations a single factory may use more than one of them at the same time or at different times. Under methods of type A the production rate of finished product P_t is increased by decreasing the average production time $\bar{p}$. Under methods of type B, the average production time is not affected by changes in the production rate and under methods of type C an increased production rate is accompanied by an increase in the average production time. The case which has been considered above falls under type B because the time profiles m_t and l_t were assumed to be constants but clearly these profiles might well depend upon the mode of operation of the factory and are not inherently independent of the production rate. Thus by treating the time profiles as variable rather than constant, the foregoing analysis generalizes to apply to all three types.

We first consider manufacturing operations which fall under type A. In this case production fluctuations are accomplished by changing the average production time. The first example of this is a factory on a constant workweek but capable of operating at a variable speed which is at any one time uniform throughout the whole process. The variable speed meat grinder is the analogy. When the speed of the production process is increased, the inflow of materials and labor increases immediately and the finished goods output increases simultaneously. There is no time delay [22] between the insertion of new material at a higher rate and the exit of finished product at the high rate, the whole process simply operates at a higher tempo. In this

[22] This, of course, does not imply that jobs pass through the process instantly—the average production time is not zero for a particular unit of material to pass through the whole process.

case the dynamic inflows to the process and the outflows from it are continually kept in balance and there is no additional accumulation of inventories in the process (equation V–1). Only the velocity of the in-process inventory has increased. In the steady state the production rate P is proportional to the speed, and the average production time $\bar{p}$ is inversely related to the speed so according to V–9 the inventory level is unaffected by the production rate. That is, an increase in P is offset exactly by a decrease in $\bar{p}$.

A second example of case A is a process which operates at a constant speed but a variable number of hours per week through the use of overtime or extra shifts. This case is the exact analogy of the meat grinder, but in neither this case nor the one discussed above is there any necessary implication that the process is continuous or utilizes discrete lots. Since the production of finished product is increased by operating the same equipment at the same speed a greater number of hours per week there is no change in the in-process inventory. Again there is continuous matching between the inflow of labor and materials and the outflow of products. Changing the hours per week that the factory operates changes the average production time since a smaller number of weeks would now be required to perform any given production job.

While type A operations are clearly quite special, they do exist and it is relevant to point out that in-process inventory is not responsive to changes in the production rate if they are accomplished by changes in the production time.

Cases which we characterize as type B have a constant production time. An example of this would be a factory with a constant workweek and production process that operate at a constant speed but have parallel sets of identical machines which can be either operated or not and in this way the production rate is varied. It is inconsequential whether the individual machines operate continuously or on discrete lots. If it is desired to increase the output, the first action is to increase the inflow of materials and labor into a previously unused set of machines. Presumably a certain length of time would be consumed before any corresponding output of finished products appears and hence there would be a dynamic increase in in-process inventory as indicated by equation V–1. Disinvestment occurs when the input of material and labor for starting new units on a set of machines is stopped but for a time the in-process units continue to arrive as finished goods output. In the steady state it is clear that the more parallel sets of machines that are operating the greater the in-process inventory as indicated by equation V–9.

A second example is a factory with a single set of machines through which all products must pass but the factory is operating well below its full capacity. The process could either be a batch process in which a larger number of batches could be handled or alternatively it could be a continuous flow process which is not kept fully charged from end to end; i.e., an additional batch of material can be run through the process. In making the assumption that the single set of machines is operating under capacity we actually require that the production lots do not interfere with each other; i.e., one lot does not in any way contribute to delaying another lot so that the production time is not influenced by the level of production.

The production rate can be increased by sending more lots through the process per period of time or alternatively by sending the same number of larger lots through or a combination of the two [23] provided that the production time is not changed. The delay between labor and materials input and corresponding output production means that dynamic inventory investment can take place and the steady state inventory is clearly related to the average number of lots in the process at any one time and this is proportional to the rate of finished production.

Hence under type B the constancy of the production time implies that inprocess inventories in the steady state are proportional to the production rate. Any dynamic changes in the output production rate require corresponding changes in labor and material inputs with a distributed lead.

All of the above cases require noninterference between different units in the production process. In general, the production time for one unit of product is influenced by the other units that are running through the factory concurrently. Indeed in many factories most of the production time of a unit is spent in a queue waiting for somebody to do something to it rather than actively being processed. In cases of type C we assume that the time for a unit of product to pass through the production process will depend upon the other units currently in the factory and hence on the production rate itself. We will not have much to say about this type of situation except to sketch out some of the problems involved.

When a factory is very lightly loaded there is in general a machine available to take on without delay any job as it appears, but as the production rate increases the situation gradually reverses until the machines are almost always busy and the jobs have to wait. Indeed to increase output further it is necessary to decrease the number and duration of machine delays in finding jobs to do. Inevitably this involves increasing the average queues of jobs ready to be done and correspondingly the in-process inventory. At low levels of production substantial increases in output can be obtained by increasing the queue inventories moderately, but at very high levels of output substantial increases in the amount of inventory in queue may be required in order to increase it appreciably. This is a complicated stochastic process which by present methods is likely to defy analysis [24] so we can say little about its inventory implications.

When machine or labor capacity is tight a reduction in machine setups can free time for increased production, but the larger lots which would be required will tend to increase the average production time since all the units in a lot may be held up at one stage until the whole lot is processed.

When we consider cost implications it is clear that where it is necessary to build up in-process inventory in order to increase the production of finished goods, the level of operation of the earlier stages would have to increase even more in order to accommodate the additional inventory investment in later stages, but this tends to make the labor input fluctuate even more than the finished goods production and presumably this is costly. It is quite clear that a great deal of work remains to be done in clarifying the issues involved and obtaining practical solutions, not to mention optimal ones.

[23] Larger lots are likely to be preferred if machine setups can be avoided by so doing.
[24] However, simulation methods using electronic computers are proving effective.

The type C form of operation is the most general and perhaps the most common one, although the simplicity and ease of control of types A and B often leads to their selection by companies if production technology permits. Where an increased amount of in-process inventory is a necessary condition for increasing the level of output, the production time including queuing delays usually increases. This is another way of saying that at usual operating levels the in-process inventory rises proportionately more than the production rate. Setting up a buffer before a machine delays the products more than the increased machine utilization speeds them up. Thus in the steady state we see using V–9 that the in-process inventory rises both with the production rate increases and with the associated increases in production time.

The necessity of intentionally building up and later liquidating in-process inventory means that the fluctuations of L_t and U_t tend to be more violent than those of finished goods production. However, the cost of fluctuating the labor input means that the optimal in-process inventory must be determined jointly with the rest of the production and employment decisions considered in section III.

As a rough hypothesis for empirical testing where operations are some mixture of types A, B, and C we would suggest very tentatively:

$$\text{V–11} \qquad I_t = C_{25} + \sum_{i=1}^{q} C_{26}(i) P_{t+i} + C_{27}\left[\sum_{i=r}^{q} P_{t+i}/(q+r+1)\right]^2.$$

The summation limit q should approximate the production time p, and r would depend upon an estimate of the decay time of the stochastic process. Under operations approximating type A, we would expect to find C_{26} and C_{27} equal to zero; under type B, we would expect C_{25} and C_{27} to equal zero. Under type C, we would expect all terms to be nonzero; the last term in conjunction with the first hopefully would approximate, at least for the steady state, the sharp nonlinear rise of in-process inventory at high levels of operations. Thus C_{27} should be positive. The coefficients $C_{26}(i)$ should reflect production smoothing considerations.

Where the production time is little longer than the decision period the summations in V–11 might run ($i=0, 1, 2$) and ($i=-r, \ldots +r$).

Presumably the pattern of materials utilization U_t which will be needed for the next section could be related similarly to the production of finished goods or alternatively derived from V–1 if the labor input were known.

When lead times are lengthened by time spent in queues, they can become so long as to constitute serious problems to the customers of manufacturers. One response by manufacturers to excessively long lead time is to increase in-process inventory so that there is a stock of semifinished items which can be adapted to customers' orders without requiring time for the complete production process. Presumably these buffers are rather closely related to the finished goods inventory and we would expect that they would be somewhat responsive to changes in forecasts of customer orders.

Fluctuations in orders received by a firm lead to fluctuations in its production, income payments and materials usage, and the latter induce fluctuations in its placement of orders which we consider now.

VI. Purchased Materials Inventory and Fluctuations in the Placement of Orders

The production process itself and any buildup of in-process inventory place demands on the inventory of purchased parts and raw materials which acts as a buffer to isolate the firm's production from fluctuations and uncertainties of deliveries by its suppliers and also to accommodate fluctuations in the production rate. In addition to its buffer function the purchased materials inventory accommodates discontinuous production and shipments in lots.

The inflow into the purchased materials inventory is regulated by placing orders on suppliers. It is an important fact that in our economy practically no penalty for fluctuations in placing orders is felt by the ordering company in spite of the fact that such fluctuations are often very costly to manufacturers as we have seen. Since these costs are not reflected back to the company placing an order, it has no incentive to avoid large fluctuations in the placement of orders and efforts to minimize their internal costs often will lead them to large fluctuations in the placement of orders. To be sure, various forms of pressure may be applied by the supplier to companies whose purchases are excessively erratic particularly in the form of reducing ancillary services. But sales departments are seldom reluctant to book an order erratic or not, and if the customer happens to be an important one, the supplier often will go to considerable trouble and expense to respond to fluctuating orders.

The fact that many firms are integrated so that the suppliers are other divisions of the same company does not substantially affect this situation since divisional operations usually are decentralized and accounting systems seldom identify the costs that are incurred by the supplying division as the result of actions taken by the consuming division.

To be sure, suppliers sometimes try to influence purchasers by offering price discounts and other inducements for purchasing in slack periods. This occurs particularly when predictable seasonal factors are involved, but dynamic pricing by suppliers which would communicate to purchasers information on desirable and undesirable times for placing orders seems to be little developed.

As the rate of utilization of purchased materials increases, economies in purchasing and shipping usually increase the desired level of purchased materials inventory. Optimal buffers of purchased materials inventory will tend to increase when the utilization rate rises, when the uncertainty of usage rate on the demand side or uncertainty of delivery time on the supply side increases (this is related to the probability distribution of forecast errors) and when an increase in lead time required by the supplier necessitates forecasts of utilization farther into the uncertain future with a correspondingly greater risk of forecast errors.

Where lead times are not "long" we would expect that changes in the usage rate would lead immediately to corresponding changes in the order rate. Also because an increase in usage rate is apt to make for larger lots and higher buffers. Hence the lots ordered are likely to be larger and the inventory levels at which orders are triggered to be raised thus leading to an increase of purchased materials inventory. Since the firm has no incentive to smooth the placement of orders, this adjustment tends to be made as quickly as individual items come up for reorder. However, the prevalence of shipments in lots introduces a distributed lag in the order response because the inventories of individual items reach their respective reordering levels at different times. The result is that fluctuations in usage rate will be amplified but with a distributed time delay.

This case might be formalized as follows. Let the desired materials inventory be $C_1+C_2\hat{U}(t)$ where $\hat{U}(t)$ is a short-term forecast of the usage rate at which materials will be drawn into the production process. Assume an exponential time delay in the adjustment of the purchased materials inventory, i.e. the rate of change of inventory is some fraction of the gap between desired and actual inventory

VI–1 $$M_t-M_{t-1}=C_{31}(C_1+C_2\hat{U}(t)-M_{t-1}),$$

where M_t is period-end purchased materials inventory. Since we are assuming a short leadtime (i.e. less than one decision period) the orders (requisitions) placed in the period t would be the sum of the forecast usage rate and the inventory investment:

VI–2 $$R_t=\hat{U}(t)+C_{31}(C_1+C_2\hat{U}(t)-M_{t-1}).$$

Although this expression is similar to some production rules which involved smoothing to avoid the costs of fluctuations, it is important to recognize that the partial inventory adjustment here is the result of a dynamic lag. The coefficient C_{31} will tend to be larger than C_{12} in the production rules indicating a faster adjustment.

The implications of "long" leadtimes can be seen most clearly for the case in which orders are placed at regular intervals of time. Consider a cost function similar to the one following II–2

VI–3 $$C=\sum_{t=0}^{N}C_5(C_1+C_2U_t-M_t)^2.$$

Here, the desired inventory of purchased materials $(C_1+C_2U_t)$ increases with the rate of utilization U_t. Deviations of inventory above or below the optimal level incur cost penalties for the firm.

We should emphasize that this cost function rests on a more detailed optimizing analysis which is not presented here (see [14] pt. C). If perfect forecasts of usage and deliveries were possible, the optimal period end inventory might be zero but lacking such foresight it is cheaper to carry buffer inventories than to run a large risk of stock outs which would disrupt the production operations. Desired inventory rests on such considerations as these.

Clearly costs in VI–3 are minimized by maintaining the following relation

VI–4 $$\hat{M}_t=C_1+C_2\hat{U}_t \qquad (t=0, 1, 2, \ldots, N)$$

between forecast inventory position and forecast usage rate.

But if there is a leadtime for deliveries the firm has no immediate control over the materials inventory. Specifically consider the case in which orders R_t placed at the beginning of period t do not result in deliveries until the beginning of the period $(t+n)$ making a total leadtime delay of n periods. Consequently the earliest period-end inventory that can be affected by present action is M_{t+n} and the relevant rule from VI–4 is

VI–5 $$\hat{M}_{t+n}=C_1+C_2\hat{U}_{t+n}.$$

The future materials inventory can be forecast using the following relation

VI–6 $$\hat{M}_{t+n}=M_{t-1}+FC_{t-1}+R_t-\hat{U}_t-\sum_{i=1}^{n}\hat{U}_{t+i}$$

where FC_{t-1} is the forward commitment of orders (placed but not yet filled) at the end of the previous period. Combining the above two equations, we can solve for a decision rule which indicates that the optimal order to be placed is

VI–7 $$R_t=-(M_{t-1}+FC_{t-1})+\hat{U}_t+\sum_{i=1}^{n}\hat{U}_{t+i}+C_1+C_2\hat{U}_{t+n}.$$

Considering the constant usage situation and pairing terms in this rule it is clear that the forward commitment would equal the usage during the leadtime n.

If we make the simplifying assumption that the same usage rate is expected for each of the periods during the leadtime, i.e.

VI–8 $$\hat{U}(t)=\hat{U}_t=\hat{U}_{t+1}= \ldots =\hat{U}_{t+n}$$

we obtain

VI–9 $$R_t=-(M_{t-1}+FC_{t-1})+(1+n+C_2)\hat{U}(t)+C_1.$$

The inventory on hand and on order may be eliminated by taking the first difference

VI–10
$$R_t-R_{t-1}=-\Big[(M_{t-1}+FC_{t-1})-(M_{t-2}+FC_{t-2})\Big] +\Big(1+n+C_2\Big)\Big(\hat{U}(t)-\hat{U}(t-1)\Big)$$

and then using the fact that inventory on hand and on order changes in response to the placement of new orders and/or usage demands, i.e.

VI–11 $$(M_t+FC_t)-(M_{t-1}+FC_{t-1})=R_t-U_t.$$

In this way VI–10 can be written as

VI–12 $$R_t=U_{t-1}+\Big(1+n+C_2\Big)\Big(\hat{U}(t)-\hat{U}(t-1)\Big)$$

or for ease of interpretation as

$$\text{VI–13}\quad R_t = \hat{U}(t) + \Big(U_{t-1} - \hat{U}(t-1)\Big) + (n + C_2)\Big(\hat{U}(t) - \hat{U}(t-1)\Big).$$

Here we can see that the orders placed in a period are equal to the currently forecast usage per period plus the amount by which actual usage in the previous period exceeded the usage that had been forecast plus an adjustment to bring the total future commitments into line with the change in the forecast usage rate. This adjustment has two components, one associated with C_2 to bring the materials inventory into line with the new forecast and one to adjust for the changed forecast of total usage during the n-period leadtime.

Here we find a rule of exactly the same mathematical form as Metzler's production rule IV–1 in which no penalty was placed on fluctuations, but the long leadtime has the very important consequence of increasing the accelerator coefficient from C_2 to (C_2+n). If the leadtime n is "large," we see that the tendency which we observed earlier in section IV for model 1 to amplify fluctuations can be increased *tremendously*!

Although the placement of orders responds to forecasts of the utilization rate, and in turn on decision policies governing production and in-process inventories which may tend to smooth their response to the fluctuations of orders received, the placement of orders is likely to have a sufficiently strong tendency to amplify fluctuations that the net effect is for orders placed to fluctuate considerably more than the orders received. Since typically orders for finished consumer goods and finished capital goods stimulate flows of orders through vertical chains of suppliers, any such tendency for order fluctuations to receive amplification at each stage has most serious consequences especially for the primary producers at the end of the chain who feel the full brunt of such a crack-the-whip process.

A sharp downward revision of the sales forecast may indicate that the existing forward commitments are already too large and R_t becomes negative which requires the cancellation of previously placed but still unfilled orders. In equation VI–13 the last two terms swamp the first one which is positive. Such a cancellation of previously placed orders is common in some industries, but is somewhat inhibited by cancellation charges which are, however, usually low.

The receipt of orders is often so volatile and the cost of responding to erratic fluctuations is so expensive that suppliers sometimes try to impose a certain smoothing on the shipments which they make to customers. However, competitive pressures by alternative suppliers severely limit such nonresponse to customer order unless the whole industry is operating at such a high level of capacity utilization that shipments are regulated by productive capacity.

The foregoing analysis of optimal periodic order placement has shown that the on-hand plus on-order purchased materials inventory is brought into line each period with the prevailing usage forecast. However, in addition to order placement we are interested in the inventory level itself and the rate of inventory investment, and we need to consider further implications of long leadtimes.

Inventory investment in the case of an exact leadtime of n periods is

$$\text{VI-14} \qquad M_t - M_{t-1} = R_{t-n} - U_t.$$

We can obtain inventory investment by substituting the order placement rule VI-12 into VI-14

$$\text{VI-15} \qquad M_t - M_{t-1} = U_{t-n-1} - U_t + (1+n+C_2)(\hat{U}(t-n) - \hat{U}(t-n-1))$$

which is very much a mixed bag. Inventory investment is the change in usage rate over a span of $(n+1)$ periods plus a large multiple of the change in forecast which occurred n periods ago.

The inventory level will be the level that was desired $(n+1)$ periods ago [25] adjusted for the accumulated utilization forecast error that has occurred since then, i.e.

$$\text{VI-16} \qquad M_t = C_1 + C_2\hat{U}(t-n) - \left[\sum_{i=0}^{n} U_{t-n+i} - (n+1)\hat{U}(t-n)\right].$$

Thus the current inventory level is strongly influenced by the forecast that was made $(n+1)$ periods ago. If large discrepancies have accumulated during the intervening periods between that forecast and actual usage, then the inventory level will deviate widely from its optimal level. Indeed large accumulated forecast errors are likely when the leadtime is long, so that material surpluses and stock out crises can be expected from time to time.

Any misestimation of the duration of the leadtime would contribute to further inventory fluctuations particularly if, as is likely, the leadtime proved to be longer than expected precisely when utilization was higher than forecast, and conversely.

In some industries, production to order requires special purchases, and hence we would also expect a relationship between the backlog of unfilled orders and the on-hand and on-order inventory of purchased materials. It is common practice when orders are received requiring special purchases, for the purchase orders to be placed immediately.

Although our analysis has focused on the manufacturing firm, the parallel should be noted between the materials purchasing function in a factory and the corresponding purchasing function in wholesale distributers and retail outlets. In all of these cases fluctuating demands (factory usage or sales to customers respectively) are placed on their purchased inventories which in turn are replenished by placing orders on their suppliers. Since in general there are no cost penalties for fluctuations in placing orders, there is no tendency toward smoothing such fluctuations that might offset the strong tendency of the inventory accelerator to amplify fluctuations. Hence we would expect to find that the whole distribution system prior to the manufacturing stages tends to amplify fluctuations in final consumer demand [26] particularly the fluctuation components that are not readily forecast.

---

[25] The extra period occurs because the forecast is made and the order placed at the beginning of the period $(t-n)$ while the inventory position is measured at the end of the period t.

[26] Some empirical work by the authors and John P. Shelton on the television industry seems to support this conclusion.

While we are not in general considering here the systems implications of firm behavior, we might note one significant relation between order fluctuations and leadtime. When the fluctuations of orders placed by many purchasing firms on a supplier happen to coincide, as they may for many reasons, the following cumulative process may operate. A flood of orders is received by the supplier and, even though he increases his production rate, the ratio of his backlog of orders to his production rate increases. This forces him to quote a longer leadtime for future orders. As is apparent from VI–9 his purchasers should in placing orders now cover themselves for anticipated usage during the longer leadtime. Thus increasing the leadtime immediately induces more orders which in turn increases the backlog of unfilled orders and the leadtime may increase still further. This cumulative process can also work in reverse leading to the collapse of orders as the leadtime is reduced.

It should be recognized, however, that where serious amplifications of the above types tend to recur, manufacturers may endeavor to base their production decisions on direct estimates of the rate of final use (e.g. retail sales) and the state of inventories in the distribution channels. This approach is not always easy to apply and is far from costless but there are indications that it has become increasingly popular in recent years.

We have concentrated on finding the implications of various cost structures for the dynamic response of the firm. However, knowledge of cost structures can also be used to estimate the costs of various fluctuations.

VII. Costs of Sales Fluctuations and Forecast Errors

The costs of economic fluctuations in the form of hardships suffered during periods of unemployment and inflation have received a great deal of attention. However, economic fluctuations also affect employed workers who oscillate between the driving pressure of long hours at one time and concern about layoffs during the slack period that follows. Business firms suffer in direct monetary terms as the result of economic fluctuations, but it is difficult to estimate these costs since they depend not only on the economic fluctuation but on what the business firms do about them.

The emergence of decision analyses offer a new approach to the study of the costs of economic fluctuations to the business firm. For example, if we have an estimate of a company's cost function and can make a quadratic approximation to it, we can determine the optimal decision rules and then using the types of forecasts or forecast relationships that are available to the company, we can simulate the performance of the company for any given pattern of sales fluctuations and determine its costs. By relating the fluctuations of the company's sales to the various patterns of national economic stability we can determine their implications for the private costs of the firm. The cost of seasonal fluctuations also can be estimated.

Similarly it would be possible to determine the costs of forecast errors or to estimate the decision performance of a company management on the basis of its historical record. Needless to say, for accurate conclusions in the last application considerable care would be required to insure the adequacy of the cost structure that is postulated.

As an illustration we present some figures drawn from an operations research study [27] for a small factory covering the years 1949 to 1953. During this period the company's sales clearly reflect the depressed general business conditions in 1949, the spurt of sales in 1950 connected with the Korean war, the inventory collapse of 1951 and the business downturn in 1953. Since only the relative costs are significant we will take the actual costs incurred by the company as 100 percent and express the other costs in relation to it. The costs included are: regular payroll, overtime, inventory holding, back order penalty, and hiring and layoff costs.

If instead of its own judgmental methods the company had used the optimal decision rule [model 5] coupled with a readily available 12-month moving average forecast, the costs would have been reduced to 79 percent. (There are a number of reasons for thinking that this improvement is too large to be representative.) Had the company had perfect foresight and used the optimal decision rule, costs would have been reduced to 72 percent. If we now hypothetically eliminate sales fluctuations altogether and substitute uniform sales, costs fall to 71 percent. Probably little weight should be placed on this particular set of data but it is suggestive of the type of analysis of costs that can be made.

The availability of a cost function makes it possible to determine the cost to the firm of sinusoidal sales fluctuations of various frequencies. In a particular case that was studied empirically (see [14] ch. 8) it was found that very low and very high frequency fluctuations could be absorbed at relatively small cost, but costs were maximum in the frequency range of one cycle per year. Such an analysis of the costs of fluctuations might make a useful contribution to the question of how much we can afford to spend on a dollar-and-cent basis in trying to improve economic stability. Clearly fluctuations of business in the macroeconomy increase the cost of doing business to individual firms even where the fluctuations are forecastable—even more so if they are not.

CONCLUDING REMARKS

The availability of decision analyses that are applicable to dynamic decisionmaking under conditions of uncertainty opens a fruitful new research approach to the dynamics of economic behavior—not limited to the production and inventory areas. Information on objectives and cost structures can contribute importantly to the specification of dynamic behavioral relationships and possibly to their estimation. This approach has been illustrated here by its application to various aspects of production, employment, and inventory decisions.

Of course, many important aspects of these decisions have not been considered. For example, we have not analyzed the important fact that inventories of all three types, finished goods, in-process and purchased materials place a demand on the working capital of the firm and obviously this imposes a limitation on aggregate inventory accumulation, nor have we considered the implications of pricing policies in moving inventory and price speculation as a motive inducing inventory fluctuations.

---

[27] Holt, C. C., Modigliani, F., and Simon, H. A., "A Linear Decision Rule for Production and Employment Scheduling," especially pp. 189–196, reprinted in "Analysis of Industrial Operations," edited by Bowman, E. H., and Fetter, R. B., Irwin, 1959.

The simplicity of using the linear decision rule analysis has much to recommend it especially in the initial research stages, but clearly more general nonlinearities and inequalities will be important to the analysis of some relationships.

As fruitful as optimizing analyses have been and still promise to be, we recognize that limitations on information, communication, and calculation of human beings in organizations place a boundary on the relevance of such analyses when their level of sophistication passes a certain point. Certainly rules of thumb, like the maintenance of constant inventory-sales ratios, explain a great deal of behavior and make direct observation of decision procedures advisable.

Hopefully the hypotheses about production, work force, and orders that have been derived here on the basis of optimizing analyses will be subjected to empirical tests and the decision models which are still in a rudimentary stage will themselves be further developed.

REFERENCES

[1] Anshen, Melvin, Charles C. Holt, and F. Modigliani, John Muth, and Herbert Simon, "Mathematics for Production Scheduling," Harvard Business Review, March 1958.

[2] Abramovitz, M., "Inventories and Business Cycles With Special Reference to Manufacturing Inventories," NBER 1950.

[3] Arrow, K. J., Karlin, S., Scarf, S., "Studies in the Mathematical Theory of Inventory and Production," Stanford University Press, 1958.

[4] Bellman, R., "Dynamic Programing," Princeton University Press, 1957.

[5] Bowman, E. R., "A New Theory About Managerial Decisionmaking," paper presented at the conference on factory scheduling, Carnegie Institute of Technology, May 1961 (unpublished manuscript, MIT).

[6] Cohen, Kalman, "Computer Models of the Shoe, Leather, Hide Sequence," Prentice Hall, 1960.

[7] Cooper, W. W., Charnes, A., Mellon, B., "A Model for Optimizing Production by Reference to Cost Surrogates," Econometrica, volume 23, pages 307–323.

[8] Darling, Paul G., "Manufacturers Inventory Investment, 1947–58: An Application of Accelerator Analysis," American Economic Review, XLIX (December 1959), pages 950–962.

[9] Duesenberry, J. S., Eckstein, O., and Fromm, G., "A Simulation of the United States Economy in Recession," Econometrica, volume 28, No. 4, October 1960.

[10] Eisner, R., and Strotz, R. H., "The Determinants of Business Investments," study paper for the Commission on Money and Credit (mimeographed, Northwestern University), especially part I, pages 49–64 and part IV, pages 1–37.

[11] Fuyrno Shozaburo, "Some Aspects of Inventory Cycles," the Review of Economics and Statistics, XLII: 203–9 (1960).

[12] Holt, Charles C., "A General Solution for Linear Decision Rules," Graduate School of Industrial Administration, Carnegie Institute of Technology (unpublished) July 1960.

[13] Holt, Charles C., and Simon, Herbert A., "Optimal Decision Rules for Production and Inventory Control," proceedings of the conference on operations research in production and inventory control, Case Institute of Technology (January 1954), pages 73–89.

[14] Holt, C. C., F. Modigliani, J. F. Muth, and H. A. Simon, "Planning Production Inventories and Work Force," Prentice Hall, 1960.

[15] Holt, C. C., and R. W. Culbertson, "Production and Inventory Control Practices and Problems as Evidenced by a Survey of 11 American Companies," 1956 Middle Atlantic Conference Transactions, American Society for Quality Control.

[16] Holt, C. C., F. Modigliani, and H. A. Simon, "Some Techniques for the Solution of Dynamic Programing Problems in Production Scheduling," American Society of Quality Control 10th Annual Meeting transactions, Montreal, Canada, June 1956.

[17] Holt, C. C., and H. A. Simon, "The Control of Inventory and Production Rates: A Survey," Journal of the Operations Research Society of America, volume 2, No. 3, August 1954.

[18] Lovell, Michael, "Manufacturers' Inventories, Sales Expectations, and the Acceleration Principle," in Econometrica, volume 29, No. 3, July 1961, reprinted as a part of his study, "Factors Determining Manufacturing Inventory Investment" in Part II of the present series of papers for the Joint Economic Committee on *Inventory Fluctuations and Economic Stabilization*.

[19] Mack, Ruth P., "Characteristics of Inventory Investment; the Aggregate and Its Parts," NBER, Studies in Income and Wealth, volume XIX, Princeton (1957).

[20] Mack, Ruth P., "Consumption and Business Fluctuations, a Case Study: The Shoe-Leather-Hide Sequence, National Bureau of Economic Research," 1958, pages 242–248.

[21] Mack, Ruth P., "The Process of Capital Formation in Inventories and the Vertical Propagation of Business Cycles," Review of Economics and Statistics, August 1953.

[22] Metzler, Lloyd A., "Factors Governing the Length of Inventory Cycles," Review of Economics and Statistics, XXIX (February 1947), pages 1–15.
[23] Metzler, Lloyd A., "The Nature and Stability of Inventory Cycles," Review of Economics and Statistics, XXIII (August 1941), pages 113–125.
[24] Mills, Edwin S., "Expectations and Undesired Inventory," Management Science, 4 (October 1957), pages 105–109.
[25] Mills, Edwin S., "Fluctuations, Uncertainty and Inventory Fluctuations," Review of Economic Studies, volume XXII(1) No. 57, 1954–55, pages 15–22.
[26] Mills, Edwin S., "The Theory of Inventory Decisions," Econometrica, volume 25, No. 2, April 1957, pages 222–238.
[27] Modigliani, F., "Business Reasons for Holding Inventories and Their Macroeconomic Implications," Studies in Income and Wealth, volume XIX, Princeton University Press (1957), pages 495–511.
[28] Modigliani, Franco, and Sauerlander, Owen H., "Economic Expectations and Plans of Firms in Relation to Short-Term Forecasting," Short-Term Economic Forecasting, Studies in Income and Wealth, volume XVII, Princeton University Press for the National Bureau of Economic Research (1955), pages 261–361.
[29] Modigliani, F., F. E. Hohn, "Production Planning Over Time and the Nature of the Expectations Horizon," Econometrica 23: 46–66 (1955).
[30] Modigliani, F., and Cohen, K. J., "The Role of Anticipations and Plans in Economic Behavior and Their Use in Economic Analysis and Forecasting," Bureau of Economic and Business Research, University of Illinois, 1961.
[31] Morehouse, N. F., Strotz, R. H., and Horwitz, S. J., "An Electric Analog Method for Investigating Problems in Dynamic Economics: Inventory Oscillations," Econometrica, volume 18, October 1950.
[32] Robinson, N. Y., "The Acceleration Principle: Department Store Inventories, 1920–56," American Economic Review, volume 49, No. 3, June 1959.
[33] Stanback, Thomas M., "Postwar Cycles in Manufacturers' Inventories," National Bureau of Economic Research, 1961, published in part I of the present series of papers for the Joint Economic Committee on Inventory Fluctuations and Economic Stabilization.
[34] Terleckyj, Nestov E., "Measure of Inventory Conditions," Technical Paper No. 8, National Industrial Conference Board, 1960, published in part II of the present series of papers for the Joint Economic Committee on Inventory Fluctuations and Economic Stabilization.
[35] Theil, H., "Economic Forecasts and Policy," North-Holland Publishing Co., Amsterdam (1958).
[36] Van de Panne, C., and Averink, G. J. Aeyelts, "Imperfect Management Decisions and Predictions, and Their Financial Implications in Dynamic Quadratic Cost Minimization," Statistica Neelandica, volume 15, 1961, pages 293–317.
[37] Whitin, T. M., "The Theory of Inventory Management" (especially pt. II), Princeton University Press, Princeton, N.J., 1953.

Appendix A

Derivation of the Optimal Linear Rule for Model 3

Since this paper has stressed the relation between cost structures and the corresponding optimal decision behavior, a simple example of the mathematical analysis for the dynamic case should help to clarify this relationship. For the rigorous mathematical details see [12].

Write the cost function with the inventory constraints as a Lagrangian function:

A–1 $$B \equiv C/C_6 = \sum_{t=0}^{N} [C_7(H_t - C_1 - C_2S_t)^2 + C_0/C_6 + C_8P_t + P_t^2 + \lambda_t(P_t - S_t - H_t + H_{t-1})],$$

where $C_7 \equiv C_5/C_6$, $C_8 \equiv C_3/C_6$ and the $\lambda_t(t=0, 1, 2, \ldots, N)$ are Lagrange multipliers.

Because of certainty equivalence, we may treat the S_t as known, and the initial conditions, i.e. variables with subscripts $t<0$, are known.

We minimize the Lagrangian function with respect to the unknown P_t, H_t and $\lambda_t(t=0, 1, 2, \ldots, N)$ by equating the first derivatives to zero:

A–2 $$\begin{aligned} \frac{\partial B}{\partial P_T} &= C_8 + 2P_T + \lambda_T = 0 \\ \frac{\partial B}{\partial H_T} &= 2C_7(H_T - C_1 - C_2S_T) - \lambda_T + \lambda_{T+1} = 0 \\ \frac{\partial B}{\partial \lambda_T} &= P_T - S_T - H_T + H_{T-1} = 0 \end{aligned}$$

$$(T=0, 1, 2, \ldots, N)$$

If we consider the three initial equations for $T=0$, we find the following four unknown variables P_0, λ_0, H_0, and λ_1 and two known variables S_0 and H_{-1}. Clearly we need to make use of the remaining equations.

To do so it is convenient to use the generating function transform since we do not require the solution for all periods, but only the action for the initial period, P_0. For a time series variable say X_t that takes a sequence of values $X_0, X_1, X_2, \ldots, X_N$, we can obtain a generating function transform defined by

A–3 $$G(X_0, X_1, X_2, \ldots X_N; z) \equiv G(X_t) \equiv \sum_{t=0}^{N} X_t z^t \equiv X(z)$$

where z is a real or complex number. Terminal conditions can be suppressed (approximately) by letting $N \to \infty$ provided that $X_t z^t \to 0$ as $t \to \infty$. This in turn is satisfied and convergence is assured if X_t is bounded and

A–4 $$|z| < 1.$$

The following shift transforms follow from the above definition:

A–5
$$G(X_{-1}, X_0, X_1, \ldots)=GX_{t-1}=X_{-1}+zX(z)$$
$$G(X_1, X_2, X_3, \ldots)=GX_{t+1}=\frac{X(z)}{z}-\frac{X_0}{z}$$
$$G(K_t)=\frac{K}{1-z}$$

where K is a constant.

If the G transformation is applied to the set of $3(N+1)$ equations in A–2, we obtain 3 equations relating the transforms.

A–6
$$\frac{C_8}{1-z}+2P(z)+\lambda(z)=0$$
$$2C_7\left[H(z)-\frac{C_1}{1-z}-C_2S)(z)\right]-\lambda(z)+\frac{\lambda(z)}{z}-\frac{\lambda_0}{z}=0$$
$$P(z)-S(z)-H(z)+H_{-1}+zH(z)=0$$

which are three equations in three unknown transforms $P(z)$, $\lambda(z)$, $H(z)$, and two unknown variables λ_0 and z.

We now use two of these equations to eliminate two of the unknown transforms and obtain a single equation in the third transform.

Eliminate $\lambda(z)$ from the first two equations of A–6:

A–7
$$2C_7\left[H(z)-\frac{C_1}{1-z}-C_2S(z)\right]+\left[\frac{C_8}{1-z}+2P(z)\right]\left[1-\frac{1}{z}\right]-\frac{\lambda_0}{z}=0$$
$$P(z)-S(z)-(1-z)H(z)+H_{-1}=0$$

Eliminate $H(z)$:

A–8
$$2C_7\left[\frac{P(z)-S(z)+H_{-1}}{1-z}\right]-2C_7\left[\frac{C_1}{1-z}+C_2S(z)\right]$$
$$+\left[\frac{C_8}{1-z}+2P(z)\right]\left[1-\frac{1}{z}\right]-\frac{\lambda_0}{z}=0.$$

Solve for $P(z)$:

A–9
$$\left[\frac{2C_7}{1-z}+2\left(\frac{z-1}{z}\right)\right]P(z)-2C_7\left[\frac{1}{1-z}+C_2\right]S(z)+\frac{2C_7}{1-z}H_{-1}$$
$$-\frac{2C_7C_1}{1-z}-\frac{C_8}{z}-\frac{\lambda_0}{z}=0.$$

The first term will vanish provided that (1) we can find a value of z that will make the bracketed expression zero and satisfy A–4, and (2) $P(z)$ is bounded. Since sales are bounded production also will be and hence $P(z)$ as well.

Equating the bracketed expression to zero yields

A–10 $$z-(C_7+2)+\frac{1}{z}=0$$

which indicates that the roots occur in reciprocal pairs, i.e.

$$z_1=\frac{1}{z_2},$$

so we can select the one, say z_1, that satisfies $A-4$:

A–11 $$z_1=\frac{C_7+2-\sqrt{C_7(C_7+4)}}{2}$$

Substituting z_t into A–9 eliminates the remaining unknown transform $P(z)$ giving

A–12 $$\frac{-\lambda_0-C_8}{2}=C_7\left[\frac{C_1z_1}{1-z_1}-\frac{z_1}{1-z_1}H_{-1}+\left(\frac{z_1}{1-z_1}+C_2z_1\right)S(z_1)\right]$$

By substituting this into the first equation of A–2 for $T=0$ and using A–10 to eliminate C_7 which is equal to $(1-z_1)^2/z_1$ we obtain

A–13 $$P_0=(1-z_1)(C_1+C_2(1-z_1)S(z_1)-H_{-1})+(1-z_1)S(z_1)$$

where

$$S(z_1)=\sum_{t=0}^{N}z_1^t\hat{S}_t,$$

N is large and z_t is given by A–11.

Using the property of certainty equivalence (see [14] ch. 6) we may replace the S_t with expected value forecasts $\hat{s}_t$. Since the same cost structure applies equally to other periods the decision rule A–13 applies to any time period t and hence may be written more generally

A–14 $$P_t=(1-z_1)\left(C_1+C_2(1-z_1)\sum_{i=0}^{N}z_1^i\hat{S}_{t+i}-H_{t-1}\right)+(1-z_1)\sum_{i=0}^{N}z_1^i\hat{S}_{t+1}.$$

Cost structures of greater complexity necessitate numerical methods but the approach to the solution is the same, see [12].

Appendix B

Cost Parameters and Decision Rules Used in Section IV

The following cost functions and corresponding decision rules were used in calculating the dynamic responses of section IV. However, in that section most of the calculations were done on an electronic computer to greater accuracy than is indicated here.

MODEL 1

Cost function:

$$C=\sum_{t=0}^{N}.0825(H_t-200.-3.0S_t)^2+0.0+51.2P_t$$

Decision rule:

$$P_t=\hat{S}_t-(H_{t-1}-200.-3.0\hat{S}_t)$$

MODEL 3

Cost function:

$$C=\sum_{t=0}^{N}.0825(H_t-200.-3.0S_t)^2+0.0+51.2P_t+0.20P_t^2$$

Decision rule:

$$P_t=\left\{\begin{array}{l}+1.140\hat{S}_t\\ \quad+.603\hat{S}_{t+1}\\ \quad+.320\hat{S}_{t+2}\\ \quad+.169\hat{S}_{t+3}\\ \quad+.090\hat{S}_{t+4}\\ \quad+.048\hat{S}_{t+5}\\ \quad+.025\hat{S}_{t+6}\\ \quad+.013\hat{S}_{t+7}\\ \quad+.007\hat{S}_{t+8}\\ \quad+.004\hat{S}_{t+9}\\ \quad+.002\hat{S}_{t+10}\\ \quad+.001\hat{S}_{t+11}\\ \quad+.001\hat{S}_{t+12}\end{array}\right\}-.47(H_{t-1}-200)$$

(Calculated from equation A–14.)

MODEL 5

Cost function: [28]

$$C=\sum_{t=0}^{N}.0825(H_t-200.-3.0S_t)^2+0.0+51.2P_t+59.0W_t+0.20(P_t-5.67W_t)^2+64.3(W_t-W_{t-1})^2$$

[28] This the real root case in [14] page 102 except that $C_8=200$ and $C_9=3.0$.

Decision rules:

$$P_t=\left\{\begin{array}{l} +1.149\hat{S}_t \\ +.607\hat{S}_{t+1} \\ +.307\hat{S}_{t+2} \\ +.145\hat{S}_{t+3} \\ +.060\hat{S}_{t+4} \\ +.018\hat{S}_{t+5} \\ +.002\hat{S}_{t+6} \\ +.010\hat{S}_{t+7} \\ +.012\hat{S}_{t+8} \\ +.011\hat{S}_{t+9} \\ +.009\hat{S}_{t+10} \\ +.007\hat{S}_{t+11} \\ +.006\hat{S}_{t+12} \\ +.004\hat{S}_{t+13} \\ +.003\hat{S}_{t+14} \\ +.002\hat{S}_{t+15} \\ +.002\hat{S}_{t+16} \\ +.001\hat{S}_{t+17} \end{array}\right\}+97.44-.464I_{t-1}+1.007W_{t-1}$$

$$W_t=.809-.0010I_{t-1}+.742W_{t-1}+\left\{\begin{array}{l} +.014\hat{S}_t \\ +.014\hat{S}_{t+1} \\ +.012\hat{S}_{t+2} \\ +.009\hat{S}_{t+3} \\ +.007\hat{S}_{t+4} \\ +.005\hat{S}_{t+5} \\ +.004\hat{S}_{t+6} \\ +.003\hat{S}_{t+7} \\ +.002\hat{S}_{t+8} \\ +.001\hat{S}_{t+9} \\ +.001\hat{S}_{t+10} \\ +.001\hat{S}_{t+11} \end{array}\right\}$$

CHANGES IN OWNERSHIP OF PURCHASED MATERIALS

BY

RUTH P. MACK

National Bureau of Economic Research

76626—61—pt. 2——5

CHANGES IN OWNERSHIP OF PURCHASED MATERIALS *[1]

The concept—ownership of materials—is intended to aid in the exploration of the complex of events comprehended under the rubric, inventory cycle. Ownership of materials combines stocks of purchased materials of manufacturers, or stock in trade of distributors, with orders for these materials that have not yet been received by the purchaser—purchase orders outstanding.

The Concept of Materials Ownership

It is of course not unusual to study outstanding or unfilled orders in connection with inventory cycles. However, the concept that I use differs from the usual one in three ways: 1. It views unfilled orders from the point of view of the buyer rather than of the seller; thus it concentrates on purchase orders outstanding rather than on unfilled sales orders. 2. It combines purchase orders outstanding with stocks of purchased materials (including stock in trade which henceforth will not be named separately) and thus deals with purchased materials on hand and on order. For many purposes, however, it is useful to separate the two components; there need be no rules on this score. 3. It excludes outstanding purchase orders of the buyer of durable producers goods. It is customary, of course, to exclude from the concept of inventories the buyer's stock of durable producers goods—the installed machinery of manufacturers or others. But the unfilled orders for these goods in the form of unfilled sales orders received by producers of durable goods are ordinarily included in the analysis of unfilled orders. Indeed, statistics on unfilled orders are dominated by this segment of the total. I exclude them because many of the influences to which they respond may be different from those that affect orders for goods intended to be resold or processed.

There are several reasons for turning to the notion of materials ownership. It seems to express how businessmen actually think about many aspects of purchasing and inventory problems. It has some interesting behavioral characteristics. It should facilitate study of how instability associated with materials purchasing and stocks is transmitted to the economy at large.

But unfortunately these propositions cannot be put to a comprehensive test on the basis of presently available information. A felt need for statistical data precedes their collection. Recommendations that certain data of these kinds be collected were made in 1955 by the Consultant Committee on Inventory Statistics organized at the request of the Joint Committee on the Economic Report, but they have not borne fruit. Clearly, the potential value of information on purchase orders needs further demonstration. The concept cuts at a tangent through much of our economic thought and therefore is not easily incorporated in its structure. The purpose of this paper is to explain

*[1] Footnotes to this paper begin on p. 85.

the economic meaning of ownership of materials and its two parts and to show how it relates to inventories and change in inventories.

These relationships are considered first with respect to broad aggregates and then as they concern the decisionmaking of the individual business. In the final section I present a few charts which show, as best it may be shown with available data, the behavior of changes in materials ownership. That it regularly leads both business cycle turns and changes in stocks is clearly indicated along with several other interesting attributes.

A Vertical Sequence Examined

Stocks are a reservoir of goods having an inlet and outlet stream. Stocks of materials, for example, have an inlet stream, goods received from suppliers, and an outlet stream, goods on which production is commencing. When receipts are larger than utilization rates, stocks of materials increase; when smaller, they decrease. The change in stock over an interval of time is equal to the inlet minus the outlet stream over the period (with minor adjustments for wastage).

Outstanding purchase orders are likewise a reservoir having an inlet and outlet stream; the inlet is new purchase orders placed with suppliers; the outlet, the physical receipt of goods. The difference between these two flows is the change in outstanding orders.

Change in the ownership of materials comprehends changes in outstanding purchase orders plus changes in stocks of purchased materials. It is the difference between the volume of new orders placed with a firm's suppliers and the volume of goods on which production is commenced; the receipts of materials cancel out, since they reduce outstanding orders and increase stocks on hand by an equal amount.

TABLE I.—*Diagram of a vertical sequence: flows of goods, changes in stocks or orders during a month, in terms of equivalent units of finished goods*

Flows of goods or orders	Case I	Case II	Stocks of goods or orders	Changes in stocks or orders: Case I	Changes in stocks or orders: Case II
CONSUMER					
1. Orders placed	100	100			
RETAILER					
2. Orders received (sales)	100	100			
3.			Unfilled sales orders	0	0
4. Shipments	100	100			
5.			Stock	0	+2
6. Receipts	100	102			
7.			Outstanding purchase orders	+4	−4
8. Orders placed	104	98			
MANUFACTURER					
9. Orders received	104	98			
10.			Unfilled sales orders	−(+4)	−(−4)
11. Shipments	100	102			
12.			Finished stock	0	+1
13. Production	100	103			
14.			In process stock	+2	−1
15. Starts	102	102			
16.			Purchased materials stock	+1	−1
17. Receipts	103	101			
18.			Outstanding purchase orders	+2	−5
19. Orders placed	105	96			
IMPORTER OR RAW MATERIALS DEALER					
20. Orders received	105	96			
21.			Unfilled sales orders	−(+2)	−(−5)
22. Shipments	103	101			
23.			Stock	−1	+1
24. Receipts	102	102			
25.			Outstanding purchase orders	0	0
26. Orders placed	102	102			
MINER, RANCHER OR FARMER					
27. Orders received	102	102			
28.			Unfilled sales orders	0	0
29. Shipments	102	102			
30.			Finished stock	0	0
31. Production	102	102			

Table I diagrams a vertical sequence of operations, one that might apply to the steps whereby cotton is converted to bath towels, steel into wood screws or cattle hide into shoes. Alternatively, it could apply to vertical steps in the economic system as a whole.

When goods move to final users, they disappear from the table. This reflects the thought that stocks of consumer goods in the hands of individuals or families, or stocks of durable capital goods in the hands of producers respond to very different influences from goods moving through the operations of production and marketing. Final product is excluded throughout this analysis.

The first two columns show vertical steps in the production, shipment or orders of goods. Each step contributes an inlet flow for one stock of goods or orders and an outlet flow for another. The changes in stocks or orders that result from differences in the volume of inlet and outlet streams are shown in the second set of columns. All figures purport to record physical units of finished goods or of constituent materials.

In order to keep the agents identified, steps that constitute virtually two sides of the same coin are listed separately. In the case of receipts and shipments there is similarity but not identity: receipts of purchases follow the same time pattern as shipments by the supplying manufacturer except for changes in time in transit (cf. lines 6 and 11). In the case of orders, there is actual identity; purchase orders placed, say, by a retailer are actually the same instruments as the sales orders of the supplying manufacturer (cf. lines 8 and 9). Consequently, changes in the outstanding orders of a customer are, precisely, changes in the unfilled orders of the suppliers. Convention records an increase of these pools of orders with a positive sign. But an increase in outstanding orders is thought of by the purchaser as an increase in ownership in the future. An increase in unfilled orders may be thought of by the supplier as a remission of ownership responsibility: it passes to his customer. Increases in stocks all represent an increase, other things the same, in ownership responsibility. To maintain consistency of thought and record, then, a change in unfilled sales orders is subtracted algebraically from an increase in stocks or in oustanding purchase orders.

This means that for a vertical sequence as a whole the net change in orders that are unfilled or outstanding is close to zero. At the finished end, we exclude changes in unfilled sales orders for goods which will be shipped to the final user, whether consumer or business purchaser of durable capital; at the crude end, the miner or farmer does not typically place purchase orders for basic stock. For intermediate orders, all the rest in the vertical sequence, outstanding and unfilled orders exactly compensate for one another.

Yet it seems clear that the presence of unfilled (or outstanding) orders and how they are changing is highly material to the power of purchasing to generate or respond to instability. Consider an example.

Assume in both cases I and II that retail sales have been rising during the course of a general cyclical expansion. The retailers in case I think that sales will continue to rise. They underestimated the rise when orders for the current season were placed and therefore they must try to get merchandise delivered swiftly. They think competitors are in a similar situation and that stocks in the pipelines are also low, and this may cause a rush for goods and some difficulty in obtaining highly desirable merchandise surely and promptly. If this occurred, prices might also rise, indeed manufacturers have been threatening that present prices would not hold except for preseason purchases. Consequently, they think it wise to increase the proportion of expected season's requirements which are ordered now for delivery in 2 or 3 months, instead of waiting for a month or so and then ordering these goods for immediate delivery. Their total current orders, therefore, those for immediate delivery and for advance delivery, are four units larger than current sales (line 8 minus 2). Their stocks have not changed (line 5) but outstanding orders have risen by four units.

The manufacturers, whose production was adjusted to current retail sales, have felt the increase in demand. New orders and unfilled orders have risen. They have responded by increasing production starts (line 15); moreover, they are buying (line 19) more than they are selling (line 9) and more than current receipts (line 17); these in turn are larger than production starts. The fact that manufacturers are buying more than they are selling, in spite of the fact that they are receiving more advance notice (more orders carrying advance delivery dates), suggests that they, like retailers, are expecting either delay in deliveries or rising prices or both. Perhaps, if it is the bath towel-cotton sequence, the published estimates of the cotton crop now look as if the crop had previously been overestimated. Supply, then, may be smaller than was formerly expected as well as demand stronger. As a result, sellers all along the line become tougher traders as buyers become more eager. In effect, short-term demand schedules shift upward and to the right; there is movement along supply schedules; but these schedules also shift in the opposite direction from demand schedules, as suppliers become less anxious to sell goods which may rise in value. If the example instead of applying to the bath towel-cotton sequence applies to the screw-steel or shoe-hide sequence, the chances of exogenous shirts in supply schedules may perhaps be somewhat less. In any event, supply schedules for crude materials have the classic upward slope. Accordingly, larger requirements are attracted to central markets from further distances, as bought scrap steel, country hides, and imported hides are induced by higher prices to augment the pseudo-byproduct supply of home-produced scrap and packer hides. But, in any case, the sensitive prices of crude materials rise.

The table shows receipts of raw material dealers responding only slowly to increased demand. They are a little higher than sales of retailers and lower than the receipts of manufacturers. Stocks of dealers decline. Perhaps if the sequence involved hides or steel, at least the orders placed by dealers would be higher because of the efforts to import hides from abroad and to collect larger quantities of scrap metal.

The example has pictured typical occurrences during an upward phase of a business cycle. Efforts to fill the pipeline augment the upward surge. Though consumer buying is increasing, the buying of retailers and manufacturers has increased more, since requirements were found to have been underestimated and delivery periods are expected to lengthen and prices to rise. At the later stages of production, swelling demand takes the form of an increase in outstanding orders and stocks of purchased material—an increase, that is, in ownership of materials. At the earlier stages, increased buying endeavor, meeting resistance of inflexible supply, presses on the price structure. The increasing orders, order backlogs, delays, and price increases support the expectation of increasing tensions and cause further buying at later stages and further price increases at earlier stages.

Contrast this picture with one which might apply in case II. Though sales have been rising, retailers had expected them to rise more and had made provisions based on these too optimistic expectations. Their stocks are quite high and have just increased two units more (line 5). A further rise in stocks does not seem warranted. Buying, it is thought, should be cautious. For, if, as seems likely, other retailers have had similar experiences, markets will soften and goods become easier to obtain on short notice. With this possibility in mind, buying is cut back substantially. Retailers' orders are cut to two units less than sales (lines 2 and 8) and total ownership declines by two units—a four-unit decline in outstanding orders minus the increase in stock on hand of two units.

Manufacturers learn of the changed opinion of retailers from the reduction in their orders as well as from trade sources. Their production starts are now equal to their shipments, but they expect shipments to decline not only because orders have, but because retail sales are lower than shipments to retailers and retailers' stocks are rumored high. Furthermore, the continued rise in materials prices has started to pinch margins so that the idea is gaining currency that prices are too high. If so, many manufacturers are thinking that you cannot make money with prices where they are because margins are too narrow; yet they fear the effect on sales of any effort to raise selling prices in line with increased costs. Accordingly, they reduce their purchasing in the hope that materials' prices will come down and goods can be picked up later at a better price. In the example, manufacturers' purchase orders are reduced even more than is their selling (lines 9 and 19), and their total ownership position is cut by two units. Their ownership of purchased materials is cut drastically—new orders are six units less than production starts. The rising finished stocks of manufacturer and dealer, and the presently undesired increase in retailers' stocks, cause buyers all along the line to be less impatient than previously; by the same token, sellers are more willing to listen. Demand and supply schedules shift. If prices do not fall, at least they cease to rise. This reinforces expectations of further weakening. Here, as in case I, output and shipments at the earlier stages are little affected as yet by the changed buying interest. Since production and shipments all along the line are still increasing, income payments to consumers are ample to support the current level of sales. But if the situation holds its present shape, production will be curtailed; then income payments associated with decisions relating to changing ownerships will be reduced even though stocks may still rise for a while.

The examples that we have been considering have concentrated, to their injury, on very limited aspects of even the central data under consideration. But in spite of their deficiencies they serve, perhaps, to suggest the meaning of the several sorts of information about stocks and orders.

Notable is the fact that in the tabular example none of the difference between case I and case II is reflected in net change in stocks. Change in stocks in all hands is +2 in both cases.

The force of buying endeavor, given the level of final demand, seems to be evidenced most immediately and accurately close to the purchasing operation. It is evidenced by changes in outstanding purchase orders and in stocks of purchased materials. Most accurately it appears in the sum of the two—changes in ownership of materials.

In the table, the difference between case I and case II is well represented by the fact that change in ownership for retailers is +4 in case I, and −2 in case II (lines 5 and 7), and for manufacturers +3 and −6 (lines 16 and 17).

Determinants of the Size of Stocks and Unfilled Orders in a Business Enterprise

Changes in aggregate ownership of materials by manufacturers or of stock in trade by distributors reflect decisions that are made in thousands of business offices. How are these decisions made? How, for example, are objectives about stocks on hand combined with those about stocks on order? How do objectives about stocks on hand and on order relate to other business decisions? How do unfilled sales orders fit into the picture? These questions call for a fresh look at the factors that influence the size of business stocks. The reasons for holding stocks need to be viewed in the setting of all business alternatives, rather than simply those alternatives that focus on stocks themselves.

The size of total stocks depends first on the amount of final product currently turned out. Second, it depends on the average speed with which goods pass through the operations that must be performed upon them. The speed, in turn, is determined by a multitude of business decisions, some of which focus on stocks and some of which focus elsewhere.

Consider a business with a given volume of sales. The size of stocks and how it changes is determined by three sorts of functions that stocks perform: (1) Providing for the necessary transit time. There is some minimum amount of time required for a series of processes to be accomplished with reasonable efficiency under a given technology; this covers economic as well as engineering considerations, and we assume that cost of carrying stocks too is to be minimized in a realistic setting. Stocks must be adequate to support, not only actual time in work, but also the necessary wait-over time associated with discontinuities in the productive process. (2) Minimizing opportunity costs with respect to the considerations governing the entrance and exit flows to a given stock reservoir. This involves the joint minimization of three sorts of costs—the cost of making the inflow conform to the outflow; the cost of making the outflow conform to the inflow; the cost of carrying the additional stocks that are required to permit varying degrees of nonconformity of inflow and outflow. When cost changes, so does the character of the reconciliation. (3) Insurance: Most decisions that affect the size of stocks involve guesses, and guesses are often wrong. It is necessary to provide a reservoir capable of absorbing that portion of the error for which the cost of absorption (carrying excess stocks) is not greater, over relevant periods of time, than is the cost of the error.

This is not the place to examine these functions systematically.[2] But I do want to say enough about them to indicate how they bear on changes in stocks and on the role that unfilled orders plays. It will simplify the discussion if stocks are thought of in terms of equivalent units of finished articles rather than in terms of dollar values. For example, the thread used in making a dress represents one unit of stock just as does the dress.

NECESSARY TRANSIT TIME

Stocks associated with transit time are a function of the rate of sales, and of the economic as well as mechanical technology of the productive or distributive process, including essential discontinuities in these processes. The discontinuities are those necessary to a joint minimization of the cost of carrying stocks and of the productive operations performed upon stocks. Examples are the need to produce in economic lot sizes and to order in economic purchase quantities and after appropriate review intervals. It is useful to confine the notion to hypothetical conditions which exclude uncertainty about sales, costs, or receipts, though expected irregularities in sales, a predetermined risk, are allowed for.[3]

Necessary transit time links stocks to expected sales in a relatively stable fashion. The basic "accelerator mechanism" features this aspect of the stock function. Quantitative aspects of the sales-stocks relation have been carefully examined and there is no need to review the literature. It seems to be agreed that the square root of sales, rather than sales proper, primarily determine the appropriate size of stocks that provide safety allowances or efficient production lots or order sizes. Of course, stocks performing these functions constitute only a portion of the total stocks associated with what I have called necessary transit time. How this portion relates to the total is shown in table II.

TABLE II.—*Hypothetical examples of changes in orders and stocks resulting from an increase in sales*

EXAMPLE I

Month	Purchase orders	Outstanding purchase orders	Stocks: Purchased materials	Stocks: In process	Stocks: Finished	Stocks: Total	Unfilled sales orders	Sales orders	Shipments
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0	10	20	10	30	10	50	0	10	10
1	18	28	10	30	10	50	1	11	10
2	11	29	10	30	10	50	2	11	10
3	11	22	18	30	10	58	3	11	10
4	11	22	11	38	10	59	4	11	10
5	11	22	11	39	10	60	5	11	10
6	11	22	11	33	17	61	6	11	16
7	11	22	11	33	11	55	0	11	11

EXAMPLE II

Month	Purchase orders	Outstanding purchase orders	Stocks: Purchased materials	Stocks: In process	Stocks: Finished	Stocks: Total	Unfilled sales orders	Sales orders	Shipments
0	10	20	10	30	16	56	0	10	10
1	18	28	10	30	15	55	0	11	11
2	11	29	10	30	14	54	0	11	11
3	11	22	18	30	13	61	0	11	11
4	11	22	11	38	12	61	0	11	11
5	11	22	11	39	11	61	0	11	11
6	11	22	11	33	17	61	0	11	11
7	11	22	11	33	17	61	0	11	11

The example in the table also deals with the replenishment period and how it affects the stock-management problem. I use the term "replenishment period" as synonymous for the purchaser with "leadtime" for the seller. Thus the replenishment period for each article that is purchased is an important element in determining outstanding purchase orders. The second determinant is the volume of purchases having each replenishment period. For unfilled sales orders, this statement should be repeated, substituting leadtime for replenishment period.

The table pictures a manufacturing establishment that sells 10 units of a product at the end of the first day of every month. Sales are believed to be invariant. Assume that after production has been completed it takes 1 month more to prepare goods for shipping. Production takes 2 months and efficient discontinuities prescribe further stocks equal to 1 month's supply. The receiving operations take 1 month and can be accomplished continuously so that only 1 month's supply is required here. Intramural time is thus 4 months; and stocks on hand constitute a 5-month's supply in terms of equivalent units of finished products. Basic raw materials cannot be obtained on demand; instead they must be purchased 2 months ahead of time—the replenishment period is 2 months. Total ownership then is 50 units of stock-on-hand and 20 units of outstanding purchase orders. The total extramural and intramural transit time is 6 months.

Now if sales did increase in spite of the assumption that they would not, customers would have to wait for 6 months before they could receive the extra goods. Shipments cannot be made from stocks now on hand, and even newly purchased materials cannot be increased until 2 months from now. There is nothing to do but order and wait.

Assume further that the sales of 11 units per month are expected to continue. Then by the time that present demand can be met, sales orders would have been one unit higher than shipments for 6 months. If sales are to be returned to an "at once" basis as soon as possible, the purchasing agent must order immediately enough to do away with the backlog of unfilled sales orders and, in addition, order enough to provide stocks which will cover the 6 months' transit time, increase in a 1-to-1 ratio with the increase in monthly sales, and cover the increase in that portion of stocks that results from the discontinuities in the production period.[4]

The table shows the provisions that are made for 8 successive months, starting with the previously adjusted level of month–0, before the increase occurred; month–7 shows the new adjusted level. Example I brings out the following facts: (1) After month–7, stocks on hand and on order have returned to their original ratio to sales—a 7-months' supply. In between time, the ratio has varied substantially. (2) The bulge starts with outstanding purchase orders and, after the period of transit has elapsed, moves on to each of the stockpiles in turn. (3) The chief adjustment is in unfilled sales orders which rise steadily over the total transit period. Customers, in this example, were willing to wait.

The second example shows an alternative way to provide for the increase in sales. Assume now that sales were thought not to be known precisely but within plus or minus one unit. For this variation a cushion had to be provided in the form of finished stocks. Assume further that it is known that if sales rise by one unit in 1 month they will not rise by an additional unit in any subsequent months in the next 6 months. The size of stocks required to provide the cushion is known from the previous example—six units. The difference between the two examples is simply that in this case finished stocks are drawn down rather than unfilled sales orders built up. Finished inventories here have the classic inverse pattern.

The two examples illustrate a type of economic technology that influences discontinuities which transit-period stocks must support—the time between when shipments must be made and purchase orders can be placed. This time is influenced by the leadtime for sales orders. If, for example, instead of selling for immediate delivery, the company only accepts orders for delivery 2 months hence, then without additional risk (assuming orders are firm) purchase orders could be placed when sales orders are written and the finished stock cushion would have had to be only four additional units. In other words, the necessary size of the cushion, other things the same, is positively associated with the replenishment period and inversely with leadtime—it is a function of intramural transit time plus replenishment period minus leadtime. However, as we shall see presently, change in the length of this period provides options that may be exercised in other ways than by trying to minimize finished stocks.

For the rest, the size of stock that is required to cover transit time is a function, given the level of sales, of (1) the actual time in operation (and note that operations are performed on purchased materials and finished goods as well as on goods in process) and (2) the discontinuities that are necessary to efficient handling of receipts, production, and sale.

These distinctions suggest how changes in business activity may be expected to affect the size of stocks associated with necessary transit time: Stocks required for the first purpose will vary in close relationship to sales. Stocks required for the second purpose typically vary less than sales. Therefore, total transit-time stocks would vary slightly less than sales. But in our example the first type is heavily dominant. Although, unfortunately there is not much known about the subject, it seems likely that for the economy as a whole the second group would be far larger than in the example. If so, the ratio to sales of transit-time stocks alone would have a further tendency to vary inversely to the level of sales.

The period between when shipments must be made and corresponding orders placed, like sales, tends to vary with business conditions. The intramural portion of it—the time between shipments of finished goods and receipts of related materials—may vary, but it is hard to say what the net impact on stocks is likely to be. On the other hand, the extramural part of it—the replenishment period—often legthens or shortens in accordance with business conditions. This influences, of course, the size of outstanding orders (and stocks if sales back up). It also affects the appropriate size of the stock cushion required to guard against the risk of running short imposed by the known vagaries of customer demand. However, changes in the replenishment period also affect the cushion required to guard against uncertainty; accordingly, we shall put off the subject for a moment.

MINIMIZING OPPORTUNITY COSTS

Carrying stock is one way of meeting business problems which can also be met in other ways. Successful overall management involves picking the least expensive way of meeting each problem.

For example, the job of selling is made less difficult, and therefore less costly in other respects, by providing customers with the advantage of swift deliveries. The job of production is made less costly, other things the same, by high and uniform levels of output. Yet if both sales and production staffs are to have their way, stocks of finished goods must be large enough to tolerate, for substantial periods of time, an entrance flow into the stock reservoir that is relatively steady and therefore differs substantially from the exit flow which responds sensitively to business conditions. This means that finished stocks tend to fluctuate inversely with sales and that they must be large enough to serve the purpose. But the size and fluctuations of finished stocks could be reduced by either or both of two means: Sales effort could be increased enough to force customers' acceptance of longer advance notice of requirements (longer leadtime on orders). Production schedules could be made to follow sales more closely.

The size of stocks and their cyclical behavior are shaped in part by decisions of this sort. Of vital importance to the way in which they are made is the relative cost, in both a narrow and a very broad sense, of these interdepartmental alternatives of minimizing total operating costs. Empirical study of these matters would be rewarding.

Only highly limited aspects of the subject are considered here—those that are of special importance in shaping short-term patterns of change in stocks and outstanding orders. Central to these questions are changes in costs that are associated with changing levels of business activity. They may be direct costs of carrying stocks or costs that represent the foregone opportunity of economizing in other departments.

Changes in costs directly associated with stocks have often been discussed. For example, change in interest rates or other carrying costs affects the optimum size of stocks. Similarly, change in the cost of placing an order affects the optimum size of both stocks and outstanding purchase orders via its impact on the optimum ordering interval or order quantity. Costs of these sorts often do, of course, fluctuate with the volume of business. No doubt, changes in interest rates, particularly, inspire changes in the amount of stock that is carried by some companies at some times, though evidence of this impact is hard to come by.

Changes in cost can also alter the relative advantage of an increase (or decrease) in a stock reservoir versus the alternative of stabilizing (or freeing) the relation between the inflow and outflow stream. These relative costs can shift as a result of changes in the level of activity or as a result of shifts in cost schedules. For example, the cost of selling often decreases (assuming no change in the cost schedules themselves) when the level of general business activity is high. Then, at current prices, and with the available staff, a company may be able to sell more goods than it can make. Choices result: the available output could be rationed by raising prices; sales effort could be reduced; orders could be accepted but delivery put off by longer leadtimes. The last alternative is a popular one.

It offers several economies: 1. Production costs may be somewhat reduced because routing can be more efficient when demand can be foretold for a longer period. 2. It reduces the financial liability for stocks, other things the same. 3. It does not imply future headaches of the sort that may attend the need to reverse alternative actions such as changes in price or personnel; changes in leadtime carry their own reversing mechanisms. 4. It reduces the cost of carrying finished stocks because shipments are evened out and therefore stocks of finished goods can be smaller and still provide for differences between shipments and production. 5. Purchasing can be done more economically by taking advantage of any one of a number of possibilities: buying just what will be required and thereby reducing stocks; buying materials at the price implied in prices charged for finished goods and thereby eliminating speculation; buying in more economical purchase quantities. 6. More advantageous purchasing may also be achieved by shifting the time when buying is done with a view to expected changes in market conditions. Shifts will be based on judgments as to whether it is better to buy nearer the earliest or the latest time that will nevertheless serve to have the goods ready to move into production when they must do so in order to meet delivery schedules. The extra foreknowledge of demand lengthens the interval between this earliest and latest moment. This point returns to our discussion concerning transit time when it was noted that a shorter interval between shipments and the time when the order for materials must be placed provided options rather than a prescribed course of action.

Another sort of change in relative costs often associated with levels of business activity concerns replenishment periods. It may be necessary, whether or not leadtime on sales orders has increased, to place purchase orders further ahead in order to forestall a rise in materials prices, a deterioration in quality, or uncertainty about whether delivery dates will be honored. Alternatives are to pay higher prices by buying from wholesalers or higher cost manufacturers. The longer replenishment period may be elected as the less costly way to meet changes in actual or expected market conditions. Often, of course, the purchaser is not offered a choice; the supplier simply stipulates the longer delivery period. Actually both elements are probably present in most situations. In any event, the immediate result is an increase in outstanding purchase orders. An increase in purchased materials stocks may be an alternative. It will in any event be a subsequent result as goods on order are received. As we saw in the example in table II, an increase in various sorts of buffer stocks, particularly finished stock cushions, may also result from the lengthened replenishment time.

Change in prices of materials expresses other shifts in cost schedules that often parallel business conditions. If prices are expected to rise, the businessman must choose whether the materials in question should be bought now, in anticipation of the rise later, or simply bought at the time dictated by necessary transit time, at the higher price, if need be. What considerations govern the choice? To simplify the picture, I will avoid the problem of maximizing advantage and discuss the conditions under which there will be at least some advantage to price-timed buying. I use the term "price-timed buying" to refer to the difference in the amount of buying that would have been done if prices were expected to be stable, and the amount of buying that is done in view of the expected change in price. The subject is particularly relevant because on the one hand of the extent to which buying, and consequently change in ownership, may be large or small depending on expectations, and on the other hand the partially self-validating potential of expectations about prices.

Assume that the purchaser is entirely sure about what the market price of a specific material will be in each of the next 6 months.

Assume also that he can fix the price by a purchase contract for any stipulated advance delivery date, and the timing of receipts is therefore unaffected by price-timed buying. Payment is made upon delivery. Accordingly, the advance purchase involves no interest or storage costs. Assume also that the material is entirely standard. Then the company should buy to cover the number of months' supply for which the present purchase price is lower than any purchase price that will prevail between now and 6 months hence. Thus, if prices are expected to rise 1 cent each month for the next 6 months, it would be advantageous under the assumption given, to buy the 6 months' requirement immediately. But suppose that prices were expected to have the following pattern for the next 6 months: $1.00, $1.01, $1.02, $1.02, $0.99, $1.01, then the purchasing agent should buy for 4 months at $1, wait until month 5 and then buy for the next 2 months at $0.99. Under the assumptions made, then, the manufacturer ought to jump immediately to the full advance position—6 months in the first case and 4 months in the second.

But, of course, the example is highly unrealistic in several respects. For one thing, a shipment schedule that is entirely unaffected by price-timed buying is not likely to be found in practice. Usually some deliveries, at least, are made; stocks are thereby increased as are the associated costs of financing, storage, and perhaps deterioration. These constitute an expense that must be set against the expected gain from early purchase. Second, a perfectly standard and homogeneous raw material is rare, so that typically it is necessary to judge how much of what sort of materials will be needed in the next span of months. The risk of guessing wrong increases as the period lengthens; it rises abruptly as seasonal peaks are surmounted. Third, the assumption of clairvoyance with respect to prices is, needless to say, unjustified; instead an important factor in determining how much to buy is how surely the price expectation is entertained. In consequence, the actual pattern of price-timed buying will reflect the process whereby opinion is formed, transmitted, gains assurance and is acted upon.

I conclude that the impact of price-timed buying, that is, of buying more or less than one otherwise would because the price of materials is expected to rise or fall, rests particularly on outstanding purchase orders. From the point of view of the purchaser, nothing else needs to change, and perhaps ideally should not, if market and institutional conditions are such that it need not. However, it is likely that stocks of purchased materials typically are also affected either immediately (e.g., in markets where goods once purchased must be shipped immediately) or later as advance orders fall due.

I have discussed three changes in costs that often move in phase with activity in the industry. Each affords choices between doing something that will affect outstanding or unfilled orders and stocks rather than something else: The choice to ration sales by extending leadtime rather than by raising prices, reducing sales staff, etc.; the choice to respond to actual or expected longer replenishment periods by buying further ahead rather than by accepting less satisfactory materials or dealing with suppliers ready to offer fast shipments; the choice to respond to an expected rise in prices by anticipating it rather than by paying the higher price when and if it occurs.

It is important to note, first, that as indicated earlier, if the first choice is resolved in favor of lengthening leadtime, the risks involved in resolving the second and third in the direction that affects ownership are correspondingly reduced. Indeed, failure to do so may actually involve the greater risk.

Second, the three choices are for other reasons too very likely to be presented to the same company at the same time. That this is the case is exceedingly important since otherwise the net effect on ownership might well be small. Although we cannot stop to evaluate the evidence,[5] casual familiarity with the behavior of markets serves to suggest that it is virtually a necessity for lengthening replenishment periods and rising materials prices (at least for the crude materials in the sequence) to occur together, and for lengthening leadtime both to support and cause these changes.

INSURANCE

Insurance against the uncertainty with which the need for goods may be foretold is an important function of stock.[6] Perhaps the major uncertainties of this sort arise from the short-term pattern of sales and of receipts of "raw" materials. The greater these uncertainties, the greater the need for stocks, other things the same. Note that uncertainty about prices is not palliated by increase in stocks and therefore is not discussed here. Again, only the broad character of the relation between changes in stocks (or in outstanding orders) and uncertainty can be sketched. Of particular interest are the changes in uncertainty that tend to be systematically associated with business conditions.

Uncertainty concerning the number and sorts of things to be sold, total volume and leadtime the same, would tend to increase the need for finished stocks in order to insure that the chances of running short shall be no greater than some stipulated figure. Whether uncertainty, thus constrained, does or does not change with the volume of sales or other business conditions is hard to say. But, as indicated in the previous section, it is clear that the constraints—sales and leadtime—do: as we have seen, increased leadtime often characterizes a period when volume of sales is high. The resultant preknowledge of sales reduces uncertainty about what items and how many of them will be sold. As a result, finished stocks to insure against the uncertainty may be reduced, other things the same. Yet the financial risk is reduced by the presence of firm orders and this factor in isolation would make a company more rather than less willing to hold stocks. What the net result of the insurance aspect would be is hard to say *a priori*.

Uncertainty about receipts of materials tend to be positively associated with the level of business. For one thing, suppliers become more independent—a usual attribute of a "sellers' market." This would counsel that the buyer hold larger stocks of purchased materials to insure meeting his production and shipment schedules. Moreover, the same uncertainty counsels an increase in outstanding purchase orders of each maturity.

All this suggests that as business improves, an increasing need for ownership to cover the insurance function of stocks is likely to be superimposed on the need associated with the expectation of longer average replenishment periods and rising prices discussed in the previous section.

Analytic Implications of Business Practices

The discussion of factors underlying change in stocks and unfilled orders abounds with questions and skimps on answers. Obviously empirical observations are greatly needed. But meaningful empirical study should cover patterns of behavior that are intelligible in terms of what we know about how people act and why. What does the analysis suggest about how this objective may be furthered?

1. Changes in stocks reflect business decisions incompletely, inaccurately, and with uneven lags. How they change is a function of the period involved in preparing goods for shipment (transit time); of specific intentions about stocks (transit time and insurance); of intentions about ownership of materials and about when purchase orders should be placed; and of all sorts of considerations that focus on materials only secondarily (minimizing opportunity costs at an interdepartmental level).

The distinction between intended and unintended stock change does not remedy the difficulty. Most unintended stock change could be prevented if the cost of doing so seemed justified.

When stocks change in a way that it seems desirable to reverse, there are many ways in which reversal can be achieved. Changes in new receipts, so important in theoretical analysis, is one of them but only one. Others are changes in selling prices, in delivery terms and services, in selling pressure, in outstanding purchase orders.

Even as an ex post record of business behavior, stocks alone reflect erratically and tardily considerations such as expectation of changing prices or replenishment periods which play an important part in their overall patterns of fluctuations.

2. Unfilled sales orders for goods bought for processing or for resale are likely to have dynamic attributes significantly different from those of goods bought for final use. It is unlikely that most of the previous discussion of the buyer's problems in connection with purchasing materials or stock-in-trade would also apply to individual families buying consumption goods or to business firms buying durable equipment. If so, it is important to investigate the two sorts of unfilled orders separately.

3. In connection with unfilled sales orders for purchased materials and stock-in-trade, there is much to be gained by focusing on the purchaser. Viewed as outstanding purchase orders, many influences that determine changes in unfilled orders and their relation to changes in stocks are seen in their relationship to the total managerial problem. Moreover, viewed as purchase orders, their relationship to the company's stock is straightforward.

True, sales orders condition many business decisions affecting stocks; they do so, in a sense, as an adjunct of sales. But there are all sorts of ambiguities in the way in which, say, an increase in unfilled sales orders affect stocks, other things the same. It reduces the need for finished stocks, and stocks in process. It reduces the risk associated with carrying stock, thus tending to increase stocks. It provides options with respect to the volume of purchased materials carried in stock and on order, the direct impact of which is more likely to fall on outstanding orders but which typically, immediately or later, also affect materials stocks on hand.

Yet time series analysis gives little expression to these ambiguities. Rates of change in unfilled sales orders seems to be the major determinant of inventory investment in two recent studies and an important one in a third.[7] The analysis of the previous section provides no operational counterpart for the empirical finding—I refer to the strength of the impact rather than to its direction which seems reasonable. Unfilled sales orders must be pushing stock around with the muscle belonging to other entities which parallel its movement.

We have learned what one of these entities is—outstanding purchase orders. At the level of aggregates, this is only partly another entity. Unfilled sales orders for materials are actually outstanding purchase orders for materials; for this segment of the total we change only the analytic frame when the same instruments are thought of in connection with the purchaser rather than the seller.

But the large bulk of unfilled sales orders for which statistics are available are orders not for materials but for capital goods typically sold to the immediate user. About 70 percent of all unfilled orders (Department of Commerce series) are those of the machinery and transportation equipment industries.[8] The patterns of variation of this group and the rest differ.

In the case of outstanding purchase orders, the relationship to stocks, particularly to stocks of purchased materials, is relatively unambiguous. Both are intimately involved in the purchasing decision—the question of what and how much and when to purchase. If sales are expected to increase, if greater stocks are required, if delivery periods are expected to lengthen or prices to rise, outstanding orders increase. Also the unfilled sales order that the company has on its books will influence its buying, and very possibly its willingness to expand its position either in terms of outstanding purchase orders or stocks of materials on hand. Supply conditions, including the delivery conditions and prices at which orders can be written, all influence and are influenced by the decision to purchase. But the point that needs to be emphasized is simply that it is in the office of the purchasing agent that problems involving sales, stocks, and unfilled orders appear as part of a single complex of decisions. If so, it is there that the causal factors can be most advantageously studied.

4. If outstanding purchase orders and stocks of materials are to be studied to maximum advantage, they also need to be studied together as ownership of materials. For their size and how it changes seems to be part of one and the same set of business decisions—decisions about how much and when to buy what materials.

In connection with expected change in replenishment periods and prices, changes in outstanding purchase orders are often of interest, sometimes to the exclusion of stocks, along, of course, with the volume of expected sales and other relevant matters. In connection with the insurance function of stocks and the minimum transit period, or in connection with purchasing to provide for changing sales itself, changes in outstanding orders are simply the vestibule through which such changes enter the business. To some extent the vestibule aspect also adheres to the element of buying that looks to market prospects.

5. Ownership objectives are more readily validated than are objectives about either stocks or outstanding orders. For this there is a technical reason. It is simply that receipts of materials, which constitute the entrance flow for stocks and the outflow for outstanding orders, cannot be accurately foretold. In addition, the outflow for stocks, production, is subject to unpredictable variation which can, if desired, be met by an immediate response of orders but not of receipts.

But there is the further reason that has to do with business objectives. We noted, in connection with price expectations particularly, that stocks on hand were a substitute (presumably a somewhat more costly one) for stocks on order but that the sum of the two could be controlled by means of changes in ordering in a way that was foreclosed to either part alone.

6. Important aspects of the impact of changes in stock on the economy are likely to take place when ownership rather than when stocks change. The excess or deficiency in buying associated with changes in ownership is likely to elicit some response in production schedules of suppliers and sensitive prices. It seems reasonable to suppose that the response could and often would occur immediately rather than at the later time when stocks might show some corresponding change.

I have argued that the study of stocks of materials on order as well as on hand is capable of providing insight into the process we call the inventory cycle. If the argument passes the first test—that of commonsense judgments of men acquainted with business problems and procedures—it needs further testing and its implications need to be explored.

Two sources of information recommend themselves. One lies in the know-how, concerns, and procedures of business enterprises. The other would utilize time series on new and unfilled sales and purchase orders in connection with the familiar data on prices, various stockpiles and related flows of production, and sales.

The Evidence of Time Series

In order to see how ownership changes along with other business conditions we naturally turn to time series. But unfortunately, as mentioned earlier, very little data on ownership are now available. If authoritative conclusions are to be reached it can only be on the basis of new data. But the collection of new basic information is troublesome and costly and it is proper to make every effort to obtain from presently available data at least some inkling of what the more appropriate information may show.

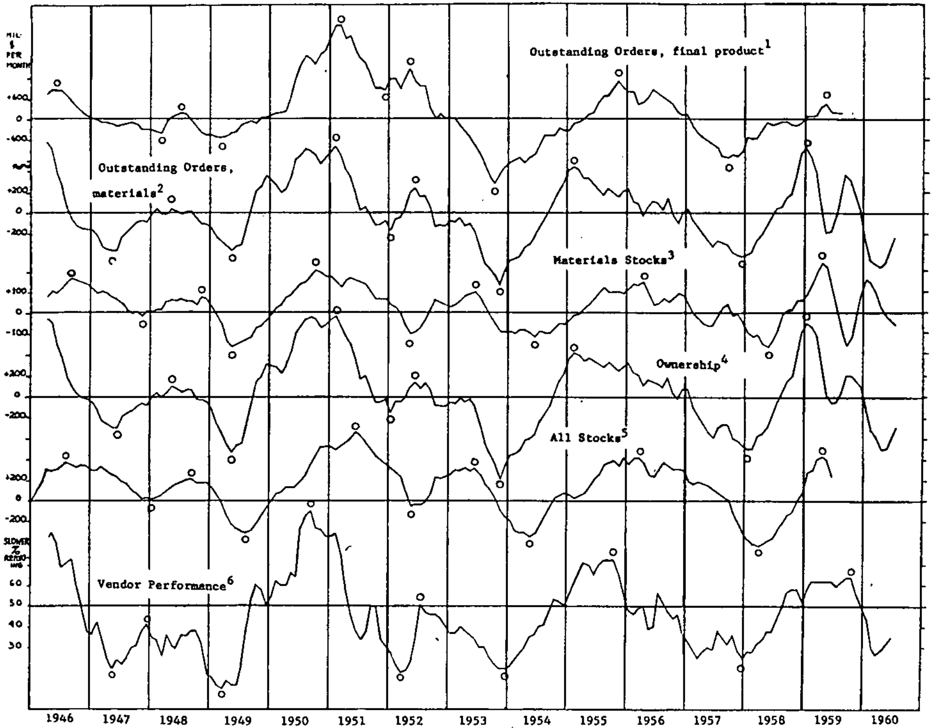
PATTERNS OF CHANGE IN OWNERSHIP

To do so is necessarily arduous and unsatisfactory. I present information based on two sets of time series: one for department stores which actually relates to materials ownership as we have defined it; the other is concocted from data for industries manufacturing durable goods and ought to approximate the information that is called for. These, in my opinion, are the most appropriate data on ownership that may now be examined.

Statistics have all been corrected for seasonal variations but not for changes in prices. There seem at the moment virtually insurmountable problems involved in deflating figures in which one wishes to study the reflection of market expectations. Most deflation techniques, in addition to all the usual shortcomings, would assume away the very problem of market oriented buying that is under study. First differences and ratios minimize the need for deflation, and I have relied heavily on these.

CHART 1

CHANGES IN STOCKS OF GOODS AND OUTSTANDING ORDERS, DURABLE GOODS INDUSTRIES, 1946-1960
(5-month averages of monthly change)



1. Unfilled orders for machinery and transportation equipment. 2. Unfilled orders for primary metals and other durables. 3. Purchased materials stocks for all durable goods industries. 4. Line 2 plus 3. 5. Total stock for all durable goods industries. 6. Diffusion index for per cent of companies reporting slower deliveries.

Chart 1 applies to durable goods. The basic data are book-value information supplied by manufacturers in the durable goods industries to the Department of Commerce. The lines of the chart depict month-to-month change (smoothed by a 5-month centered moving average) for stipulated stocks of goods and of outstanding orders.

The Department obtains information on sales orders only. Among these we need to distinguish between those orders placed by final users and those placed by firms who intend to process or resell the goods they buy. The separation is made on an industry basis. I assume that the large majority of sales orders received by the primary metals and other durables [9] industries are sold to processors or distributors and therefore are purchase orders for materials to be used in the manufacture of durable goods. Purchasers might be distributors or manufacturers in any of the durable-goods industries—intermediate or final processors. In contrast, the machinery and transportation equipment industries are excluded because they sell directly to final users.[10]

Comparison between the first two lines on the chart indicates that outstanding purchase orders for final product and for materials show some interesting differences in their patterns of month-to-month rates of change.[11] Though there are differences in timing, they are not systematic; they both have strong leads relative to business cycle turns but themselves follow no consistent priority.[12] But outstanding orders for the final product of the durable goods industries wax and wane in strong, largely uninterrupted cyclical rhythms, whereas materials seem more prone to additional movements.

The first occurred during 1946 and early 1947. This was a period when the postwar recession was scheduled to appear. Seventy-four percent of the executives responsing to questionnaires by Fortune magazine, distributed in May of 1947, said that they expected business conditions to worsen. The second extra movement consisted of a readjustment in 1951–52 after the excited efforts to amass goods during the early stages of the Korean war. Though not entirely absent in the heavy equipment industries, it is far clearer in the intermediate products.

The third line in the chart exhibits changes in stocks of purchased materials. These, in line with our definitions, are those of all durable-goods industries. We note a general similarity in the contours of investment in stocks and in outstanding purchase orders though the latter lead persistently; indeed at times of strikes (e.g., 1952 and 1959) the relationship is perhaps more nearly inverse. Most of the leads are between 4 and 6 months. These observations are based on comparison between the months when matching turns occurred in each series.[13] The months when turns have been recognized are indicated by the circles on the chart. Table III gives the frequency distributions of leads or lags.

TABLE III.—*Frequency of leads or lags of various lengths in rates of change of specified data for durable goods industries, 1946–59* [1]

Timing of—	Compared to—	Leads (months)				Synchronous, 0	Lags (months)			
		7+	5 and 6	3 and 4	1 and 2		2 and 1	3 and 4	5 and 6	7 and over
Outstanding orders, materials.	Outstanding orders, final product.	1	------	1	2	------	3	1	------	------
Do	Materials stocks	3	5	1	------	1	------	1	------	------
Ownership	Outstanding orders, materials.	------	------	------	------	8	7	------	------	1
Do	Materials stocks	1	1	2	6	------	------	------	------	------
Do	All stocks	3	1	4	3	------	------	------	------	------
Materials stocks	do	1	------	1	1	1	7	------	------	------

[1] Leads or lags are between matched turns in 5-month centered moving averages of month-to-month change. Turns are indicated by 0 above (peaks) and below (troughs) the lines in chart 1.

The length of the leads raises puzzling and fascinating questions. Clearly this does not reflect a simple vestibule relationship; replenishment periods are far shorter than this. Suggested is some building-up process which we do not as yet know how to describe.

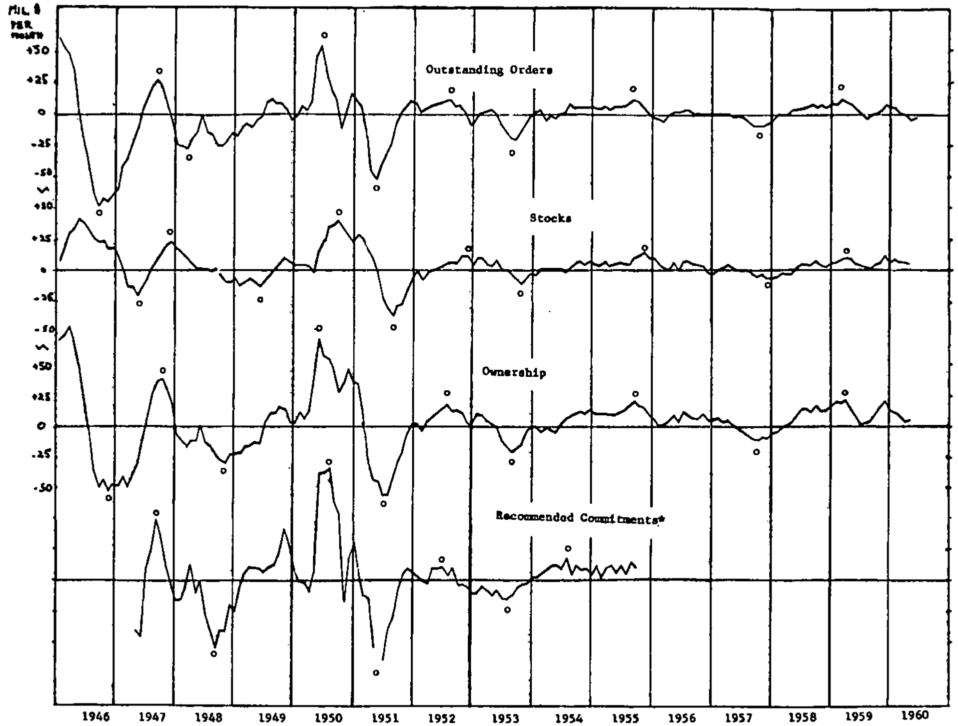
The fourth line in the chart shows change in ownership, the sum of the two preceding lines. For ownership proper, the purchased materials component (materials for all durable goods industries) is slightly less than half of the total, and unfilled orders (for the two intermediate industry groups) a bit more than half. And our figures doubtless underestimate the relative importance of outstanding orders.[14] Nevertheless, rates of change (the data depicted) are heavily dominated by the orders, rather than the stock, component. Note that stocks are plotted twice the scale used for orders. Peaks and troughs in ownership tend to synchronize with those of orders or to follow them after 1 or 2 months, whereas, they regularly lead those of purchased stocks.

The fifth line in the chart (the sixth is discussed at a later point) shows change in all stocks of the durable goods industries. Rates of change in all stocks is quite similar to that of the purchased materials segment, which is little over a third of the total for stocks proper.

Change in ownership regularly leads change in total inventory investment. Seven of the matched turns lead by 2 to 4 months, whereas, four have longer leads. This is interesting primarily because of the broad similarity in the contours of the two series.[15]

CHART 2

CHANGES IN STOCKS OF GOODS AND OUTSTANDING ORDERS, DEPARTMENT STORES, 1946-1960
(5-month averages of monthly change)



*Number of months of expected sales that should be covered by commitments for merchandise; the data plotted are 5-month averages of month-to-month change in recommendations.

The next chart covers about the same ground as the previous one except that it concerns department stores that report merchandising data to the Federal Reserve Board. In this case information refers, precisely, to ownership as we have defined it.

For department stores, there are virtually no unfilled sales orders hence the mate to the first line of chart 1 is not shown; it would simply be a horizontal line at the zero point on the vertical axis. Changes in unfilled purchase orders for stock in trade, on the other hand, fluctuate substantially. They appear at the top and can be compared with changes in stocks just below. These are total stocks which, by our definitions, are included in ownership for distributors.

In spite of the exceedingly heavy seasonal movements that characterize all department store statistics and the difficulties that correcting for them can introduce, there appears here, too, to be broad parallelism in investment in outstanding orders and in stocks. Likewise, there are occasional hints of inverse association such as during the period of adjustment to postwar decontrols. Outstanding orders proper, which are about half the size of stocks proper, have about the same range of fluctuation in rates of month-to-month change. (The vertical scales are the same so a simple visual comparison of amplitudes may be made.) Here, as for durable goods, changes in outstanding orders lead those of stocks; here, too, leads seem longer than the vestibule function alone would suggest. For the 11 matched turns, the frequencies of leads of various durations are as follows: 2 months, 1; 3 or 4 months, 7; 8 months and longer, 3. Change in ownership is again more similar to change in outstanding orders than in stocks. Three turns are synchronous and ownership lags orders by 1 or 2 months at the other seven turns. On the other hand, ownership leads changes in stocks (1 or 2 months, six times; 3 or 4 months, two times; and over 7 months, seven times).

FACTORS INFLUENCING CHANGES IN OWNERSHIP

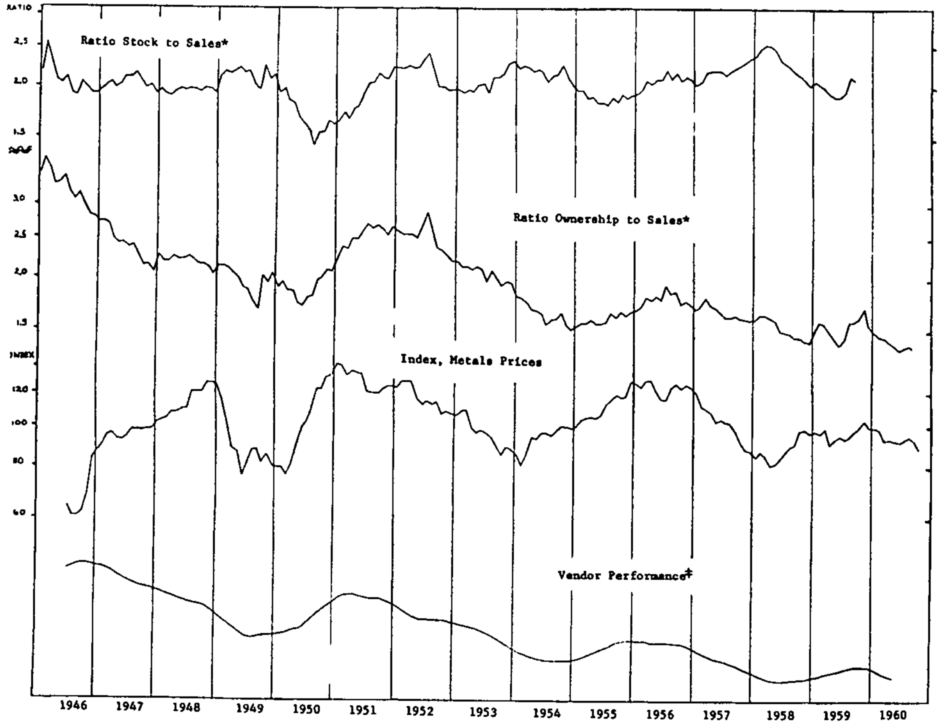
Our theoretical analysis suggested that changes in ownership should reflect changes in customer's requirements as evidenced by sales or perhaps sales orders; it should also reflect changes in replenishment periods and in expectations about prices. These are perhaps the most important determinants though many others have relevance.

This might be taken to mean, that were it possible to produce time series representing each of these variables, in a multiple correlation frame, ownership would be "explained" by the other three. Certainly it would be interesting to try an experiment of this sort, though I have not done so. However, a word of caution is in order. Two-way causality would rage within the analysis. It is hard to say whether changes in sensitive prices and replenishment periods are more cause or more effect of changes in ownership. Likewise, change in supply conditions and in short-term demand play tag with one another. The term "feedback" is far too orderly and tame a word to describe this degree of causal runaround.

Nevertheless, a first step is to look for parallelism, and I do so in two stages. Ownership is expressed as a ratio to sales. This enforces an inappropriately rigorous form on the character of the relation, but it can at least produce a negative result. Because we presume that

Chart 3

TURNOVER RATIOS, SENSITIVE PRICES AND REPLENISHMENT PERIODS FOR DURABLE GOODS INDUSTRIES, 1946-1960



*End of month stock divided by sales during the month.
‡Cumulated diffusion index for per cent of companies reporting slower deliveries.

ownership may be rapidly adjusted to desired levels, there is no reason to use sales of earlier rather than current months. The second step is to see whether the ratio changes in a way that might reflect the influence of the other two variables.

The second line in chart 3 is the ratio of ownership at the end of a month to sales of all durable goods during the month. Unlike the ratio of stocks to sales, shown at the top of the chart, which tends to move inversely in the neighborhood of cyclical peaks and troughs, the ownership ratio generally conforms to cyclical patterns—synchronously, and with a lead as well as sometimes with a lag. The ratio, in short, behaves in a way which could reasonably be intended by business executives.

Do these intentions appear to involve guesses about materials prices or information about the length of replenishment periods? As evidence about price expectations, we can view only actual prices, but I select those most sensitive to business conditions. Current spot market prices for metals (five are included in the index) would certainly be examined by someone trying to guess what any sort of simple processed metal is likely to cost in the future. Except for the first few years when the aftermath of war disturbed both prices and procurement, there appears to be a rather striking parallelism in the two sets of data. The fact that prices lead is interesting and demands study.[16]

Concerning the length of replenishment periods, there is one very useful body of information. It consists of reports by members of the Purchasing Agents Association of Chicago on whether deliveries of major materials are getting slower. The charted series is a cumulation of a diffusion index for these data.[17] A diffusion index for the components of an aggregate tends to resemble first differences of the aggregate itself, thus the cumulation produces a facsimile of the aggregate.[18] Here again we seem to view a sequence of events that could well be influencing ownership. The postdecontrol decline in ownership, unreflected in prices, is evidenced in the increasing speed with which deliveries are made.

This association can be viewed in another way in chart 1, where the percent of purchasing agents reporting slower deliveries (bottom line) is compared with the change in ownership (fourth line). In view of the entirely independent source of these two series, the parallelism is, I believe, quite impressive.

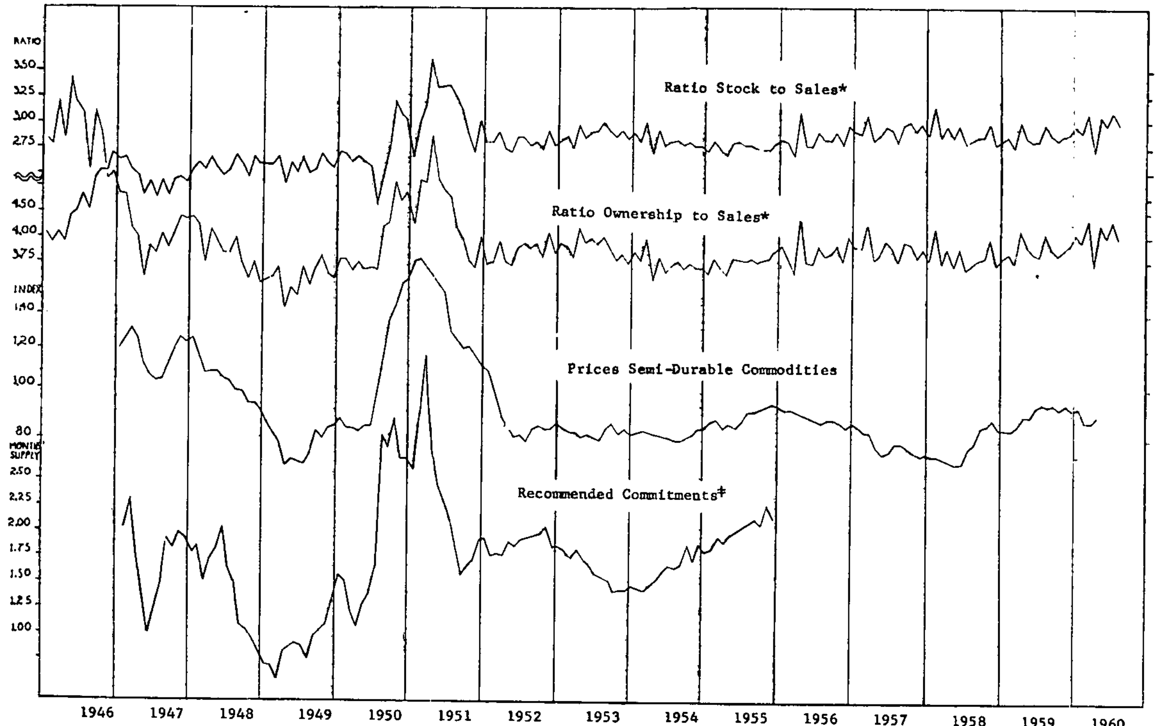
The same question concerning evidence of factors influencing ownership may be put to the department store data. Chart 4 starts with the ratios to sales on the assumption, by way of a first approximation, that stocks are intended to have a uniform overall relationship to sales, other things the same.

The stock-sales ratio exhibits an irregular relationship to business conditions, showing faint and tardy evidence of business peaks in 1948, 1953, and 1957 or troughs in 1949, 1954, and early 1958. The ratio of ownership to sales, on the other hand, seems to respond promptly and often with a lead to business cycles as well as to the phantom postwar recession and the Korean boom.

By hypothesis, movements in the ratio reflect reactions to changing replenishment periods and expected prices. Though the things that department stores buy are well removed from raw materials market, spot market prices are likely to contribute to the formation of expectations about the prices (and more subtle quality changes) of goods

CHART 4

TURNOVER RATIOS, SENSITIVE PRICES AND REPLENISHMENT PERIODS FOR DEPARTMENT STORES, 1946-1960



*End of month stocks divided by sales during the month.
‡Number of months of expected sales that should be covered by commitments for merchandise.

that stores buy. Accordingly the third line presents a combined index for the 9 semidurable industrial materials for which spot prices are reported.[19] The first 5 years show a synchronous parallelism of sorts. Perhaps most impressive is the difference in the shape of the Korean boom as reflected in both the ownership ratio of department stores and semidurable goods prices on the one hand, and the materials ownership ratio and prices in the durable goods industries, on the other hand. (Compare charts 3 and 4.) But the price index seems to give no clue to motives for change in ownership (which, incidentally, did not appear to be very strong) during the half a dozen years from 1952 through 1957. Could changes in replenishment periods be responsible?

There are as far as I know no data on actual replenishment periods for department stores. However, a business consultant firm provides retail buying guides to its customers. These guides stipulate the number of weeks requirements that should be held on hand and on order for each of a large number of departments.[20] These data, kindly supplied in connection with an earlier paper, were averaged for all departments.[21] The recommended position varied from about three-quarters of a month's requirements early in 1949 to almost 3½ months at the peak of the Korean war scarcities. The material is reproduced as the last line in chart 4. The picture is not inconsistent with the thought that the recommendations were based on knowledge about changes in "vendor performance" and that this actually does influence retailer's buying policy.

Again the first differences in the hypothetical replenishment period (smoothed by a 5-month centered average) are shown as the last line of chart 2 in order that the gross association with change in ownership may be inspected.

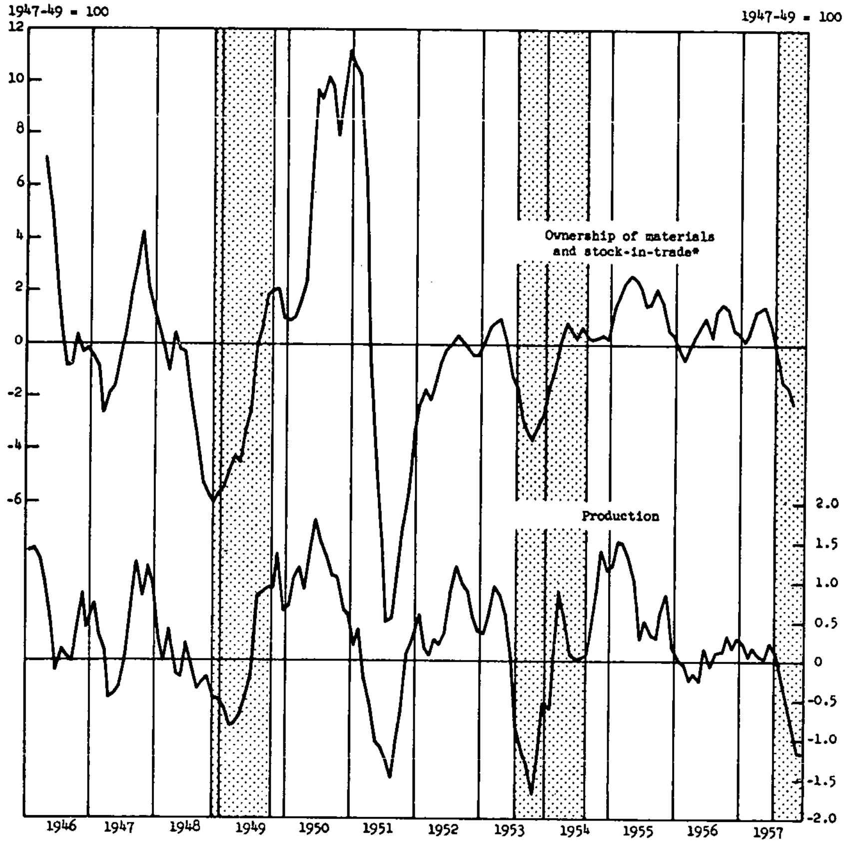
A FRAGMENT CONCERNING OTHER CAUSAL ASSOCIATIONS

I have suggested that inventory cycles may have a major impact on the economy at the time that changes in ownership rather than in stocks occur. The reason is that the additional buying which the change in stocks on hand and on order represents is interpreted by the people who receive the orders as warranting more production than would otherwise be undertaken. Since only increments are involved, the expected association would be between rates of change in the suppliers' production and rates of change in customers' materials-ownership. Incidentally, the causal association could also have reverse elements, insofar as increasing rates of output mean slower deliveries and therefore longer leadtimes (replenishment periods for customers). The problem is to develop a set of figures in which customers' materials-ownership can be compared with suppliers' production.

For the durable goods group nothing of this sort can be accomplished.[22] For department stores, if sales of durable goods are ignored, the output of all semidurable goods manufacturing industries can be compared with change in ownership for department stores plus change in ownership, for, hypothetically, those semidurable goods manufacturers whose suppliers are other semidurable goods manufacturers. This concept is shown, probably without gross violation, in chart 5, and there compared with month-to-month change in production of semidurable goods manufacturers.[23]

CHART 5

CHANGES IN OWNERSHIP AND PRODUCTION OF SEMIDURABLE GOODS, 1946-1957
(5 month averages of monthly change)



*Estimated ownership of stock-in-trade by retailers and of materials by manufacturers.

The far greater amplitude of ownership than of production during the Korean crisis is at least in part a function of the sharp rise in prices, which, though implicitly present in the book value of ownership, is absent in the production data which are in physical terms. For the rest, we see again a parallelism of minor as well as major ups and downs without a systematic tendency for either series to precede the other. The data seem quite agreeable to the notion that they are partly tied by causal association.

THE NEED FOR DATA

I conclude that the major propositions that were developed in the previous section are not seriously challenged by the time series that it has been possible to display. Quite the contrary. Changes in outstanding purchase orders, alone and in combination with stocks of purchased materials precede changes in stocks, exhibiting also interesting sorts of parallelism and inversion. Ownership itself seems to reflect intended behavior in its association to sales, replenishment periods, and prices. Figures give a whiff of suggested interrelation between changes in materials-ownership of a customer and changes in the production schedules of his supplier.

Obviously these impressions need to be checked and sharpened. The first test is the commonsense judgment of others about the way the time series behave. The reader has no doubt himself decided whether the ups and downs of the time series in his opinion suggest noteworthy association. Of course, it would have been useful, had it also been possible to view, drawn to the same scale, the multitude of other series for which judgments about noteworthy association would in my opinion be more doubtful or negative. A further check, particularly in connection with the relation between ownership, sales, replenishment periods, and prices might be provided by correlation analysis though there are hazards here, too.[24]

But however fully developed these aggregative data may be, firm conclusions require more appropriate figures. Data on purchase orders are required. They need to be supplied by the same companies for which statistics on sales orders, stocks by stage of manufacture, shipments, production, and receipts are available. Statistics for major commodity sequences must be studied individually; in this connection, it would be desirable to investigate the feasibility of developing from-whom-to-whom tabulations for orders. Physical volume data are needed. These are not largely new conclusions. The Consultant Committee on Inventory Statistics, organized at the request of the Subcommittee on Economic Statistics of the Joint Committee on the Economic Report, made recommendations along these lines in their report of November 1955: Recommendations Nos. 21 and 22 (pp. 84 and 86) proposed that data on sales, stocks, and outstanding orders be published in seasonally corrected form for major departments of department stores. Recommendations 29 and 30 (pp. 94 and 96) were addressed to methods of obtaining adequate physical volume statistics. Recommendation 32 (pp. 99, 100) urged the need for "physical quantity data on stocks and on production, shipments, and orders, for important sequences of commodities through the various stages of production and distribution." It is noteworthy that Japanese statistics now include data on orders classified by the industry originating the order.

Finally, statistics alone are only one tool with which to carve comprehension. The question I raise concerns business practices. These practices themselves, and the objectives and problems that lie behind them, need to be studied with the aid of business executives.

The problem to be examined is deeper than appears at first glance. True, there is an evident need to explore whether expectations about prices and replenishment periods, along with expectations about sales, influence procurement, and how important these considerations are judged to be. But the deeper questions, and ones highly relevant to efforts to subdue inventory fluctuations, concern how these influences, if present, act and are acted upon.

Let me illustrate. Thomas Stanback, in a report included in part I of the present series of papers for the Joint Economic Committee, expertly explores inventory statistics and finds himself pushed toward recognizing changing supply conditions as an important governor of inventory behavior. My discussion here has emphasized short-term changes in demand—changes, that is, in the willingness to buy stipulated quantities of goods at stipulated prices (shifts in "demand schedules"). Apparently we are both impressed by the need to look toward changing market conditions if inventory fluctuations are to be understood. But one emphasizes short-term changes in supply and the other demand. What does this apparent inconsistency mean? It means, I think, that both views are correct and both seriously incomplete.

It is clear that changes in market conditions occur. In addition, these changes, and expectations of further changes, make sellers coy and buyers eager at the same time. Why then is there not an explosive struggle for goods and skyrocketing prices followed swiftly by equally abrupt reversals and plummeting markets?

The answer must lie in the character of a process which is social as well as economic. It concerns how opinion seeps from product to product, from firm to firm, within and among industries; how opinions, gaining assurance, result in actions; how actions themselves change the external facts to which opinion looks. There are self-limiting as well as self-generating aspects to this process. There are time durations that must be studied. Raw materials for such study fill the air and files of every business office.

Time series likewise provide essential raw materials. Signs of the process itself and of its implicit time dimensions seem to flicker even in the inept time series that we have viewed. The signals appear in the extra-cycles in ownership and the too long and irregular leads. More appropriate statistics would afford a clearer view.

If market conditions and expectations, and the process whereby they diffuse, cumulate, and reverse, play an important part in economic destabilization of various sorts, we need to embark on gathering information about buying and selling with at least a small part of the vigor with which, starting years ago, we have gathered information about income and product.

Notes

1. The paper reports some of the results of a research project conducted at the National Bureau of Economic Research. I am indebted to Arthur F. Burns and Geoffrey Moore for far-reaching constructive help in dealing with the problems here presented.

2. The three functions include the factors emphasized in other classifications. Thomas M. Whiten in "The Theory of Inventory Management" (Princeton University Press, 1953, p. 850), for example, says the "principal causes of inventories * * * are expected changes in cost or demand functions, discontinuities in the rates of ordering, production, sales, and the uncertainty of demand." My "transit time" covers his "discontinuities" and fills in, I think, the omitted in-process time. My "opportunity costs" covers the changes in cost or demand functions that he mentions and includes others. His "uncertainty of demand" is part of my "insurance" element, although I include uncertainty of supply and of production as well.

3. I make the usual distinction between risk, which is involved in a small number of drawings from a known probability distribution, and uncertainty as to the distribution itself.

4. The table simplifies by assuming a constant ratio instead of using the square root optimizing formula.

5. The logic of business management problems seems to prescribe these relationships in a fairly obvious fashion but evidence on the point is scattered. This is a major theme of my study, "Consumption and Business Fluctuation; a Case Study of the Shoe, Leather, Hide Sequence" (National Bureau of Economic Research, 1956). Perhaps the most interesting evidence on the nature of these interconnections is in editorial sections of trade journals. The Commercial and Financial Chronicle has sections on "The Financial Situation" and "The State of Trade," "Commercial Epitome" that carry information back into the 19th

century. Business Week's section on "The Outlook" and the Journal of the National Association of Purchasing Agents are other particularly fruitful sources.

6. For the distinction between risk and uncertainty, see Note 3.

7. The first two are: Michael Lovell, "Manufacturers' Inventories, Sales Expectations and the Acceleration Principle;" change in unfilled orders are used to explain change in stocks of purchased materials and goods in process; and Measures of Inventory Conditions (Technical Paper No. 8, National Industrial Conference Board, 1960) by Nestor E. Terleckyj; the actual variable used is the ratio of new orders to sales; this differs from the simple difference, which is change in unfilled orders, merely by introducing a proportional element; calculations refer to all stocks of manufacturers and distributors. The third study is "Manufacturers' Inventory Investment, 1947–58," American Economic Review, December 1959, pp. 950–962, by Paul G. Darling.

8. See the following:

Manufacturers' unfilled orders, end of July, 1953–60

End of July	Total		Machinery and transportation equipment industries			Other industries	
	Book value (million)	Index 1960=100	Book value (million)	Index 1960=100	Percent of total	Book value (million)	Index 1960=100
1953	$70,472	148	$51,674	157	73.3	$18,798	172
1954	46,158	97	35,173	107	76.2	10,985	74
1955	47,765	100	33,227	101	69.6	14,538	98
1956	59,057	124	42,005	128	71.1	17,052	115
1957	56,146	118	40,752	124	72.6	15,394	104
1958	44,036	92	33,114	101	75.2	10,922	74
1959	47,254	99	33,571	102	71.0	13,683	92
1960	47,690	100	32,860	100	68.9	14,830	100

9. "Other durables" include instruments, lumber, furniture, stone, clay and glass, ordnance, and miscellaneous durables. Ordnance ought not be included but cannot be removed.

10. Fabricated metals is a mixed category which, accordingly, has been excluded. Nondurable materials purchased by durable goods manufacturers is also not covered.

11. Any period-to-period (e.g., month-to-month) change in stocks is, of course, often spoken of as "inventory investment." Indiscriminately, I refer to it also as "change," or "rates of change." In connection with the charts, 5-month-moving averages of monthly change is implied. These terms refer to changes in outstanding orders (actually a stock of orders for goods) in exactly the same way as they refer to changes in stocks of goods.

12. All the leads for materials relative to final product occur at peaks, whatever this may signify.

13. The turns are not necessarily only those that would be deemed turns in "specific cycles" by the Bureau, though most usually they would be. Movements directly associated with the steel strike in 1959 have been bypassed. In matching turns I aim at comparing the turning points of like fluctuations.

14. Stocks are theoretically complete. But our series on outstanding purchase orders have known exclusions: Orders are not included at all for materials bought from other than manufacturers. Moreover, though there are probably no sales orders for final product included in primary metals and other durables, there are doubtless many purchase orders for materials, such as those in fabricated metals industries and some nondurable goods industries, including those of distributors, that are excluded from our series.

15. The two share purchased materials which for the data I use constitute about two-fifths of ownership and about one-third of total stocks. (There is no reason to consider this a factual description of true ownership.) But as noted earlier, investment in ownership is far more strongly influenced by its outstanding orders than by its stock component. The same five waves appear in both series and with generally similar relative severity. Also, most of what has been said about ownership and total stocks would apply only a little less sharply to outstanding purchase orders for materials alone and total stocks. More serious

qualification would be needed in describing the association between change in outstanding orders for final product (line 1) and change in total stocks.

16. The amount of potential gain from price-timed ownership is a function of the rate of change in prices rather than the level of prices proper. The rate of change in prices would lead prices proper and thus, insofar as it influenced behavior, tend to cause the relevant series on actual prices to lead still more. There is a complex line of reasoning which offers an explanation for this but it cannot be discussed here; moreover, there are several points of fact that would have to be determined.

17. The percent of companies reporting slower deliveries are tabulated (adding one-half of those reporting no change) and the percentages are cumulated. The very large majority of reporting companies are in the durable goods industries.

18. A diffusion index is identical in movement to a first difference series in which all components always increase and decrease by an identical amount. In practice, though this condition is of course never met, the two sorts of data typically are broadly similar. By the same token, a cumulation of a diffusion index tends to approximate aggregate data proper.

19. They include wastepaper, burlap, cotton, hides, print cloth, rosin, rubber, tallow, and wool tops.

20. Requirements are defined as sales plus intended change in stocks.

21. R. P. Mack and Victor Zarnowitz, "Cause and Consequence of Changes in Retailers' Buying," A.E.R., March 1957. We were indebted to A. W. Zelomek and Robert C. Shook of the International Statistical Bureau for the figures and permission to publish them.

22. Since materials ownership for the first producers in the vertical sequence is not included in our calculation, neither should production. Yet if we exclude production of the primary metals producers, there is little left that refers to suppliers, which is called for here, rather than to purchasers.

23. Total ownership consists (in 1947–48) of one-half ownership for department stores and one-half materials ownership for semidurable goods industries. The latter is composed of—

(1) Purchased materials stocks for apparel, textiles, paper, printing and publishing, and leather. Each industry was seasonally corrected separately.

(2) Unfilled sales orders for nondurable goods industries reporting unfilled orders to OBE, reduced to exclude value added by manufacture (to convert unfilled sales orders to an estimate of outstanding purchase orders). Industries are those mentioned above with the exception of apparel for which unfilled orders are not reported. Data for each industry was seasonally corrected separately.

Production of semidurable goods manufacturers is the Federal Reserve Board's index of production of nondurables minus the component for food. Data are seasonally adjusted and smoothed by a 5-month centered average of month-to-month change.

24. In addition to the price problem discussed above, the wide fluctuations during the Korean war period may exert a disproportionate influence on regression coefficients. The meaning of the results requires, therefore, special examination in connection with these data.

THE IMPACT OF CREDIT COST AND AVAILABILITY ON INVENTORY INVESTMENT

BY

PAUL F. MCGOULDRICK

Capital Market Section,
Board of Governors, Federal Reserve System

THE IMPACT OF CREDIT COST AND AVAILABILITY ON INVENTORY INVESTMENT*

The proposition that a rise in interest rates and reduced availability of bank credit tend to limit investment in business inventories may command general acceptance, but there is a wide area of disagreement with respect to the magnitude of this effect. Two poles of opinion may be distinguished. On the one hand, it is held that credit conditions have had very little effect on postwar inventory fluctuations. Because of the dominance of price and sales expectations in shaping business inventory decisions, business demands for funds to finance inventories have been highly insensitive to interest rates. Furthermore, it is argued that the large proportion of investment needs that can be met by retained earnings and the drawing down of liquid assets has made business relatively immune to changes in credit availability. A number of business opinion surveys and regression analyses are adduced in support of these conclusions.

At the opposite pole of opinion, it is argued that interest rates are—or have recently become—important in business calculations at the margin of inventory investment. Moreover, limited availability of bank credit during periods of restraint influences inventory investment that may not be sensitive to interest cost. Although credit cost and availability may affect only a minority of firms, the decisions of this minority can have a significant effect on aggregate inventory investment. To accomplish this, credit cost and availability do not have to reach extreme degrees of tightness; relatively gentle alterations in terms of lending can have noticeable effects on inventory outlays. Supporting these conclusions are quantitative studies embracing both English and American experience during the 1950's.

The present paper will first summarize and appraise theoretical and other nonstatistical arguments that support a significant linkage between credit conditions and inventory accumulation. It will then examine the theoretical case against this proposition. Finally, the various statistical studies bearing on this proposition will be reviewed and appraised.[1]

Theoretical case for credit effects on inventory investment

Costs of carrying inventories.—The classic argument for a tight link between bank credit and inventory investment is that of Hawtrey.[2] In his opinion, credit regulation has very little effect on plant and equipment outlays because of technical lags between investment planning and realization and the narrow range of fluctuations in long-term rates of interest. On the other hand, short-term interest rates fluctuate more widely over the cycle and have substantial

* The views expressed in this paper do not necessarily represent those of the Board of Governors of the Federal Reserve System.

[1] Throughout this study credit conditions are considered at the level of the business borrower. This means, for example, that lags between monetary policy decisions and changes in credit conditions are not discussed.

[2] Ralph Hawtrey, "Capital and Employment," 2d ed., 1952.

effects on the cost of carrying inventories, even though they are only a minor element in the total cost of doing business:

An item of cost that is negligible in an enterprise as a whole may be substantial in relation to a limited section of its activities. An increase of postage charges is not likely to make all the difference between the flotation of an enterprise and its abandonment, but it may cause a substantial reduction in the number of letters and circulars dispatched. Similarly, an increase in the rate of interest on bank advances will not result in a contract being refused that would otherwise have been accepted, but it may materially affect the buying program for the replenishment of stocks.[3]

In an example used by Hawtrey, the total cost of carrying copper inventories, including interest, averages between 6 and 10 percent of its selling price. Even taking the upper figure as one minimizing the interest rate element of cost, a 1-point shift in the interest rate would mean a 10 percent rise or fall in total carrying cost of copper inventories. And, he argues, interest rates are usually a higher fraction of carrying costs of manufactured goods than of those of wholesaled raw materials.

Hawtrey contends further that businessmen are able to respond promptly to changes in carrying costs due to shifts in interest rates, since—

the sacrifice of convenience (in holding smaller inventories) is very small. If he has been accustomed to order 6 weeks' supply of a commodity at a time, it will be very little additional trouble or expense to order 3 weeks' supply only.[4]

In appraising the Hawtrey theory, critics have pointed to two weaknesses of fact. The additional inconvenience and noninterest costs from lowering inventories may not be small as Hawtrey argues. Lower inventories increase frequency of purchases, and this reduces economies of large lot buying, to which raw material inventories are particularly subject. Secondly, Abramovitz has argued that inventories of goods in process and to some extent raw materials are determined rather rigidly by production needs, and therefore variations in interest costs have extremely little effect on these components of total inventories. Although inventories of finished goods are not tied to production requirements, they are tied to sales and the desire to stabilize output.[5] At the extreme, inventories of perishable or style goods with wide seasonal fluctations in production and/or sales may be almost completely insensitive to interest rates.[6]

These criticisms need not invalidate Hawtrey's basic conclusions. Inventory fluctuations need not be large relative to the average level of stocks held in order to be significant in dollar volume. An argument along these lines is presented by White: [7]

Twenty percent per annum of the value of the inventory is a typical figure for marginal inventory carrying costs (interest, handling, taxes, insurance, deterioration, etc.). A 2-percentage-point rise in interest rates would raise the carrying costs to 22 percent—only a 10-percent increase—and would raise the square root of the carrying costs by only 4½ percent, thus indicating only a 4½-percent reduction in inventories (according to the formula cited above).[8] It will be noted, however, that with actual inventories (excluding goods in process) at $74 [billion]

[3] Ibid., p. 107.
[4] Ibid, p. 74.
[5] Moses Abramovitz, "Inventories and Business Cycles," National Bureau of Economic Research, 1950, chs. 8, 9, and 11.
[6] Many of the rigidities cited by Abramovitz are due to seasonal fluctuations in output and sales but have no bearing on cyclical swings in inventories discussed here.
[7] William H. White, "Inventory Investment and the Rate of Interest," Banca Nazionale del Lavoro Quarterly Review No. 57, June 1961, pp. 151-152.
[8] The formula cited is given in a study of formula for inventory management when interest and other costs vary (T. M. Whitin, "Theory of Inventory Management," Princeton, 1953). As an approximation, Whitin states that the optimum purchase quantity varies inversely with the square root of carrying charges of inventories.

as of January 1959, the rational 4½-percent reduction would be an appreciable factor in the U.S. business cycle. A $3 [billion] reduction in these inventories concentrated in a 6-month period would be equivalent to a one-sixth reduction in gross business investment.

This is not to say that in fact bank lending rates have recently fluctuated by as much as 2 percentage points over 6 months (or indeed, over the cycle). Such movements have occurred in Great Britain, but not in the United States. Also, the pertinent interest cost associated with much inventory investment is probably an opportunity cost, since over one-half of inventory investment has in recent years been financed from retained earnings, depreciation allowances, and reductions of liquid assets. But an opportunity cost is just as real as a money cost: as an alternative to inventory accumulation firms can always invest funds in Treasury bills, tax anticipation certificates or time deposits, or they can repay bank debt.[9]

What the example quoted by White demonstrates is that profit-maximizing behavior can result in substantial inventory investment or disinvestment in response to changes in interest rates. This still follows when the Whitin formulae (of which the shortcut quoted is only a simplifying approximation) recognizes and includes the increase in noninterest costs associated with a reduction in inventories (for example, loss of large lot discounts). Hawtrey's conclusion remains valid even when noninterest costs dampen the effects of interest rate changes.

Whether most businessmen tend to maximize profits in this way is a question of fact, not of theory. White[10] has contended that many large American firms have recently adopted inventory management formulae analogous to those derived by Whitin and that this movement is still going on. But whether this changeover has been massive enough to affect current total investment appreciably is another question.

Incidentally, the rapid introduction of these inventory management techniques can cause once-for-all reductions in inventories, which complicate interpretation of inventory statistics. In a period of credit restraint, such once-for-all reduction would exaggerate the normal influence of credit conditions on inventory investment.

Financing inventory investment.—A substantial share of inventory investment has traditionally been financed by short-term borrowing. Furthermore, borrowing from banks to finance extensions of trade credit to customers may well end up by financing inventory accumulation by the latter. Thus the connection is a close one, even if not watertight. Apart from interest cost effects, therefore, changes in the availability of bank loans to business may have a significant impact on inventory accumulation.

Against this presumption, it has frequently been said or implied that credit availability and interest rates can have only a tiny effect on inventory cycles because most inventories are financed by equity and long-term capital.[11] For example, in March 1961 the total of short-term and long-term bank loans to corporate manufacturers amounted to only 23 percent of the book value of their inventories, and short-term loans alone were only 14 percent of inventory book value.

[9] It might also be noted that interest rates on Treasury bills have fluctuated much more than bank lending rates over recent cycles. So changes in opportunity cost can have more leverage on inventory policy than changes in bank lending rates.

[10] White, op. cit., p. 174.

[11] See, for example, "Staff Report on Employment, Growth, and Price Levels," Joint Economic Committee, December 1959, pp. 390–391.

The relevant comparison in studying inventory investment, however, is between changes in inventories and changes in bank loans. Since much inventory working capital is a permanent component of total capital, it is not surprising that this component should be financed, like fixed capital, from stockholders' investment, retained earnings, and long-term nonbank borrowing. However, this permanent component is irrelevant for decisions at the margin of investment. What is relevant is the availability of funds to finance that portion of inventories which fluctuates over the seasons and over the business cycle.

In September 1955, for example, short-term bank loans were only 10 percent of the book value of inventories, in corporate manufacturing. But in the following 2 years the increase in short-term bank loans was equal to 25 percent of the $10.8 billion increase in these inventories. In the business upswing from June 1958 to June 1960, the ratio between the changes in short-term loans and inventories, in corporate manufacturing, was about the same (24 percent).

As in the case of interest rate effects, it is not possible to determine on theoretical grounds whether or not inventory investment is in fact influenced by changes in credit availability. But some characteristics of inventory investment provide reasons to believe that inventory investment is capable of responding promptly and/or sizably to change in credit conditions.

Characteristics of inventory investment.—(1) Inventories are highly divisible compared with plant and equipment. Much fixed capital investment is of an all-or-nothing character or, at least, is very "lumpy." The divisibility of inventory investment facilitates small adjustments appropriate to moderate changes in credit cost and availability.

(2) The amount of inventory accumulation in any time period will ordinarily be only a small proportion of total materials purchased or output of finished goods in that time period. As a result, a small reduction in purchases of materials during a period would be sufficient to reduce intended investment in materials inventories by a substantial amount.

(3) Businessmen can reduce inventories, thus investing negatively, while gross investment in fixed capital cannot fall below zero. This facilitates larger changes in inventory investment than in plant and equipment outlays, especially in recessions and early recoveries.

(4) As compared with fixed capital outlays, decisions to alter inventory investment can be made effective promptly. The acquisition of capital goods involves considerable lags between decisions to purchase and the final realization of those decisions.[12] The average lag between inventory decisions and/or output and purchases of materials is relatively short.[13]

[12] Frank deLeeuw, forthcoming paper on the demand for capital goods by manufacturers in Econometrica and Thomas Mayer, "Plant and Equipment Lead Times," Journal of Business of the University of Chicago, April 1960, pp. 127–132 (quoted by deLeeuw).

[13] In investigating lags of inventory investment in manufacturing after cyclical turning points, Abramovitz (op. cit.) examined technical relationships between output and inventories of purchase materials and goods in process for the years 1920–46. He found that goods in process tended either to move synchronously with output or to lead output slightly. Raw materials stocks moved with lags of between 2 and 3 months behind output, due to delays between order placements and receipt of shipments. Stocks of finished goods had significantly longer lags behind cyclical turning points owing to "the interval between input and output in manufacturing establishments . . . continuing uncertainty about the course of sales . . . reluctance from the point of view of personnel policy to hire workers who may have to be laid off within a short time or lay off workers who may have to be hired soon . . . and to gain other real or supposed benefits from stabilizing output." Post-1946 data on finished goods inventories in manufacturing show no lag longer than 6 to 7 months after cyclical points, and the decision lag is necessarily shorter than the lag behind sales.

(5) Although intended alterations in inventory investment may respond promptly and sizably to credit tightness or ease, the published inventory statistics tend to conceal this. Intended changes in inventory investment have effects on output, employment, and consumption which bring with them unintended inventory changes in the opposite direction.[14] A planned reduction of inventories by one firm tends to reduce sales of its suppliers, and this in turn causes unintended increases in inventories of suppliers. Even if all firms together attempt inventory reductions, they cannot escape this offset. The resulting decline in purchases and incomes causes a drop in sales and hence a partial offset in the form of unintended increase in aggregate inventories. (A similar offsetting effect occurs in the case of planned increases in inventories.) Thus, any intended response of inventory investment to shifts in credit cost and/or availability is always diminished, never exaggerated, in the necessarily ex post inventory statistics.[15] This makes it particularly difficult to establish quantitative relationships between credit conditions and inventory investment (as, indeed, between nonfinancial variables and inventory outlays).

Although unintended changes in inventories tend to offset intended changes, almost simultaneously, intended changes will ultimately have their full effect. The Metzler model [16] tracing these effects under simplified conditions has been the foundation of much subsequent theory and empirical research on inventory investment. In brief, it shows that a rise (fall) in intended investment always causes a rise (fall) in money income, even when feedback effects via employment and income are so strong as to offset an intended change almost completely. Put another way, a change in intended investment always tends to cause a change of income in the same direction, while the countervailing change in unintended investment acts only to shift part of this induced change in income into the future. When intended accumulation is offset by unintended disinvestment, businessmen will respond by increasing output or orders in an attempt to attain their original inventory targets.

Theoretical case against credit effects on inventory investment

Corporate liquidity.—Corporations entered the post-World War II period in a state of high liquidity. As a result, they have been able to draw down or refrain from increasing cash and other liquid assets as a means of financing inventory investment when bank credit was costly or difficult to come by. This, in turn, has made their inventory policies relatively immune to variations in credit conditions.

Corporate holdings of liquid assets (cash and U.S. Government securities) have increased considerably less than the income and product of the corporate sector. If, for example, liquid assets of nonfinancial corporations had risen as much between 1949 and 1959 as national income originating in corporate business, they would have been $20 billion higher in 1959. This $20 billion may be regarded as having been released by the willingness and ability of corporations to become less liquid. Over the same period, corporate inventory investment amounted on balance to almost $28 billion. In a sense, therefore, the running down of liquidity can be interpreted as having

[14] Cf. Abramovitz, op. cit., p. 13.

[15] This is a unique property of inventory investment. If plant and equipment outlays are diminished for any reason, the feedback effects via reductions in employment, income, and consumption tend to reinforce the original decrease.

[16] Lloyd Metzler, "The Nature and Stability of Inventory Cycles," Review of Economic Statistics, August 1941.

directly or indirectly financed a substantial proportion of inventory accumulation.[17] Furthermore, although the statistical association is not close for all years, the release of liquid assets tended to be large in years when inventory investment was large, as the following table shows:

	Cash and Government securities "released" [1]	Inventory investment
	(In millions of dollars)	
1950	+0.6	+4.8
1951	+6.1	+8.6
1952	+0.3	+2.2
1953	+6.3	+0.8
1954	−1.4	−1.9
1955	+1.5	+4.9
1956	+2.9	+4.9
1957	+4.0	+0.6
1958	−5.0	−2.6
1959	+5.0	+5.3
Total, 1950–59	+20.3	+27.6

[1] This equals the (percentage change in national income originating in corporate business times liquid assets at the beginning of the year) minus the actual change in liquid assets. This "release" is a virtual one. In a majority of years, corporations added to their holdings of cash and U.S. Government securities. But except for years where the sign in col. 1 is negative, they did not add as much as would have been required to hold liquid assets at their 1949 relationship to income in corporate business.

Source: Calculated from SEC estimates of current assets and liabilities of all U.S. corporations except banks, insurance companies, and savings and loan associations.

It is likely that there is considerably less scope now for reducing liquidity than there was 10 years ago. Thus, even if it is true that corporate inventory investment in the postwar years was relatively independent of changing credit conditions because of the availability of liquid assets, this independence is weaker now.

Trade credit.—By extending trade credit, firms with a surplus of internal funds or ample liquid assets or better access to external credit can indirectly finance inventory accumulation by customer firms in a weaker financial position. From 1949 to 1959, accounts receivable of nonfinancial corporations rose from $34 to $86 billion, and net receivables (measuring the net extension of trade credit to other sectors but omitting net trade credit extension within the corporate sector) increased from $8 to $24 billion.[18]

A broad and positive association exists in manufacturing between cyclical swings in total inventories and in net trade credit extended per dollar of sales.[19] Trade credit has therefore provided a gentle and rather steady offset to bank credit stringency for customers of manufacturing firms. However, trade credit can only finance the cost of materials, not value added to inventories.

Insensitivity to interest costs.—The interest cost of doing business is only a tiny fraction of labor or material cost or of profits. Even in the recession year 1958, interest payments were only 14 percent of profits after tax earned by manufacturing and trade corporations, which account for the bulk of inventory investment. It may be argued therefore that the interest cost of funds is so small, relatively,

[17] Part of this virtual release of liquid assets may have financed acquisition of other assets, including plant and equipment and accounts receivable.
[18] Flow-of-funds estimates based on SEC data.
[19] The ratio of net receivables to sales also had an upward trend from 1949 to 1959.

that it enters the thinking of only a few businessmen making decisions about inventory investment.

Abramovitz, for example, is skeptical of an interest rate effect of any magnitude. Suppose that the interest rate were 6 percent at the peak of business activity. Then, suppose that the rate fell to 4.8 percent at the trough. On a quarterly basis, this is only a fall from 1.5 to 1.2 percent of the cost of carrying inventories with borrowed money—

> a difference altogether too small to affect a manufacturer's calculations about the profitability of carrying the additional stock necessary to do extra business.[20]

As Abramovitz states, the relevant profit rate to compare with interest cost is the ratio of profit earned on the marginal inventory holdings to the cost or selling price of these holdings. However, as Nicholas Kaldor emphasizes,[21] the latter rates of profit are probably much smaller than the average profit rate on all assets. Because inventory investment generally involves much less risk than fixed capital investment, we should expect inventory investment to be carried to the point where marginal earnings are much smaller than either average or marginal earnings on fixed capital assets.

Logically, therefore, the appropriate profit rate for inventory changes may well be low enough so that small alterations in interest rates could have a decided effect on inventory holdings. This would be so for the firm of economic theory: all-knowing and sensitive to even tiny changes in costs. But in actuality most firms may be led by businessmen insensitive to small variations in interest rates, at least during the short periods appropriate for cyclical analysis.

Effect of corporate tax.—A related argument is that the corporate income tax results in an approximate halving of interest costs at the margin. Thus, a 2-point change in interest rate causes only a 1-point change with respect to its impact on after-tax profits.

This is a widely used argument in favor of interest insensitivity of business investment. If businessmen actually believe and act on this reasoning, its relevance cannot be denied. But, as stated, the argument is not consistent with the assumption that businesses act so as to maximize profits. A firm has no incentive to behave differently with respect to costs because of the corporate income tax. For the tax affects not only interest and other costs but also gross earnings. As a result, the relative impact of changes in interest rates is not blunted by the existence of corporate income taxes. Therefore the reaction of inventory investment to interest rate changes should be the same whether or not an income tax is imposed, assuming profit-maximizing behavior.

"Real" rate of interest.—The argument is frequently advanced that rising prices have offset much of the deterrent effect of rising interest costs. In other words, "real" interest rates have risen less and have been much lower than nominal interest rates.[22]

---

[20] Abramovitz, op. cit., pp. 115–116.

[21] Radcliffe Committee, Minutes of Evidence, p. 715.

[22] If the annual money rate of interest is r and the annual percentage change in prices is p, the real rate of interest is $\frac{1+r}{1+p}-1$.

For example, nominal and "real" interest rates were as follows in the years 1954–57:

	Nominal rate [1]	Percent rate of price change [2]	Real rate
1954	3.6	+0.4	3.1
1955	3.7	−0.3	4.1
1956	4.2	+2.1	2.1
1957	4.6	+3.1	1.5

[1] Bank rate on short-term business loans.
[2] Weighted change in GNP implicit price deflator for durable and nondurable consumer goods from fourth quarter to fourth quarter. The weights are proportions of inventories of durable and nondurable goods in manufacturing and trade.

As can be seen, the real rate declined after 1955 and was lower in 1957 than in 1954, even though credit conditions were more stringent in 1957.

It may be questioned whether decisions to invest have been influenced by the prevailing real rates. More relevant are expectations regarding prices at the time funds are borrowed; unless prices are advancing rapidly, expectations of increases and decreases in prices will largely offset each other. However, it cannot be denied that during most of the 1950's, real rates were below nominal rates and general awareness of this tended to blunt cost restraints on borrowing.

Availability of bank loans.—A final consideration working against an effect of changing credit conditions on inventory investment is that the availability of business loans may be relatively insensitive to changing credit conditions, over the cycle. Banks are likely to regard the extension of these loans as their major function. Changes in their reserve positions are therefore reflected in the availability of other types of loans (real estate, security, etc.) and in their holdings of securities.[23]

In other words, the supply of bank loans to finance inventory investment may be quite elastic and may not shift cyclically with credit conditions. If this is so, it would not be surprising if empirical data did not show a strong statistical relationship between inventory investment and credit conditions.

Insofar as banks maintain the availability of business loans by selling Government securities in a period of credit restraint, it might be argued that rising interest rates on these securities is reflected in bank lending rates and that this in turn would deter borrowing. However, cyclical increases in yields on Government securities have been reflected only partly in bank lending rates over the whole postwar period. Secondly, to the extent that bank loan interest rates increase, the effect on demand for loans may be small, quite apart from changing price expectations and expected "real" rates of interest. For many borrowers the limit on their borrowing consists of credit availability, not its cost. In other words, they would be willing to borrow more even if the interest rate was somewhat higher. Unless the increase in interest rates is quite large, effective demand for loans may not decline at all.

[23] Cf. "Staff Report on Employment, Growth, and Price Levels," Joint Economic Committee, December 1959, ch. 9, pp. 390–391.

Empirical studies

Statistical and measurement problems.—The empirical findings reviewed in this section must be taken with reserve. The immediately preceding discussion of intended and unintended inventory changes points to one source of difficulty in the interpretation of inventory statistics. Another is that regression studies have had to use time series that are highly correlated with each other because of common trend and cyclical movements. This is illustrated in the following table of simple correlation coefficients that emerged in a Federal Reserve study of durable manufacturing inventories from 1952 to early 1961.

Matrix of simple correlation coefficients

	Current inventory level (book value) (1)	Lagged sales (2)	Lagged inventories (book value) (3)	Interest rate on business loans (lagged) (4)	Current sales (5)
1	+1.00				
2	+.88	+1.00			
3	+.96	+.78	+1.00		
4	+.90	+.81	+.91	+1.00	
5	+.76	+.86	+.60	+.68	+1.00

Source: Inventories and sales, seasonally adjusted, Department of Commerce. All lags are for 1 quarter.

If more than one of these variables is utilized as an explanatory (independent) variable in a multiple regression study, the contribution of the second (or third, etc.) such variable in explaining the total movement of inventories will appear to be small (a problem of multicollinearity). Furthermore the regression coefficients are apt to have a wide margin of uncertainty.

An even greater problem in regression studies is that of specifying the direction of influence (an identification problem). For example, although changes in sales are frequently taken as an explanatory variable for changes in inventories, changes in inventories also affect sales directly and immediately. Most regression studies have attempted to overcome this problem by using lagged values for the independent variables; that is, they have attempted to explain changes in one variable by past changes in other variables. But this procedure begs the question of anticipations. For example, present inventory investment can be said to have had no influence on past sales only if the consequences of present investment or disinvestment were not foreseen to some extent and did not thus condition sales in the past. For periods as short as one quarter, used frequently in regression studies, this is doubtful. (An anticipated steel strike is an example of a situation in which anticipated future changes in inventory influence current sales.)

This problem is particularly serious in studies involving interest rates and indexes of credit availability. As the preceding table shows, interest rates almost invariably rise when inventories rise and fall when inventories fall. But this scarcely means that higher interest rates encourage the holding of more inventories. The interest rate is the price of loanable funds, and it tends to rise and fall with inventories and the demand for funds to finance them. Furthermore, interest rates and credit availability tend to influence "independent" and "dependent" variables in the same direction; e.g., they also affect fixed capital outlays, which in turn influence new orders, frequently taken as an independent variable. As a result, the effects of interest rate and credit availability changes are difficult to isolate statistically.

Surveys among businessmen designed to throw light on their behavior with respect to inventory investment are not subject to these drawbacks and are very useful in estimating the direction and rough magnitudes of various influences on business decisions. But among the problems peculiar to surveys are: the difficulty of formulating unambiguous questions, nonresponse, the proper weights to give individual answers, differences among survey periods in business and credit conditions (including differential movements among industries and different sizes of firms) and frequent unawareness among respondents of their real motives for action. In particular, it is difficult in a survey to isolate changes at the margin, yet, as indicated above, it is these that are crucial.

A central difficulty for all empirical studies is that of measuring credit availability effects. Availability has many dimensions, including financial strength of borrowers, riskiness of loans, the purposes for which loans will be used by borrowers, industry of borrower and cycles of sales and borrower liquidity peculiar to these, the type and subjectively judged adequacy of bank secondary reserves, and whether banks hold net excess reserves or are indebted, on the whole, to the Federal Reserve System. To the extent that availability changes cannot be and/or are not measured in quantitative studies, effects of credit conditions will tend to be understated. This would not hold if some sensitive interest rate such as that on Treasury bills is a properly weighted index of both credit cost and availability, but this assumption is highly doubtful.

Weaknesses in the inventory data themselves are covered by another study in the present series of papers for the Joint Economic Committee and need not be reviewed here. Not enough is known about the causes of error in inventory measurement to say with any definiteness whether they are mostly random or not. Even benchmark revisions of the SEC corporate and GNP inventory series are not necessarily reliable.[24] Until more information on errors in inventory statistics is gathered, it is permissible to assume that these may be treated as random (not systematic). But the fragility of this assumption must be kept in mind in interpreting results of regression or other studies.

Regression study by Solow et al.—In an unpublished memorandum prepared for the Commission on Money and Credit, Robert Solow, E. Cary Brown, John Karaken, and Robert Ando found by means of multiple regression analysis that the interest rate on short-term bank loans had a significant effect on inventory investment in manufacturing. Specifically, the statistical finding was that a 1 percentage point rise in the interest rate on bank loans during a quarter would, other things remaining equal, reduce the book value of inventories by $1.15 billion [25] during the following quarter. Ultimately, if no other factors intrude and sales and interest rates do not change further, inventories would fall by $4.86 billion. Although the process would be slow, being about two-thirds completed after 1 year, inventory investment

[24] See on this point, "Statistics of Business Inventories" (report of Consultant Committee on Inventory Statistics, organized by the Board of Governors of the Federal Reserve System), November 1955, pp. 41-42.

[25] The net regression coefficient for the interest rate is statistically significant, at the 5-percent level.

would have its maximum reduction right after the interest rate change.[26] In subsequent quarters, induced declines in investment would be successively less, tapering off ultimately to zero.

Basically, the model assumes that the level of inventories is adjusted to the level of estimated sales (generating investment and disinvestment in inventories), but only with a lag. An outstanding virtue is the explicit recognition of unintended inventory accumulation as a (possibly) systematic component of total investment over the cycle. Another virtue for present purposes is its test of whether interest rates affect the desired level of inventories.

The hypothesis tested may be stated as follows: The level of inventories is equal to the previous quarter level plus intended and unintended inventory investment. Intended investment is determined by the difference between last quarter's inventories and the level businesses regard as most appropriate for the current quarter, though only a part of this difference is made up currently. Unintended inventory accumulation (or reduction) is assumed to be equal to difference between the expected and the realized level of sales in the current period. The rate of interest is brought in as a factor influencing (negatively) the desired level of inventories corresponding to any given level of sales.

Solow et al. also tested for the effect of credit availability on inventories, substituting an availability index (maximum potential earning assets of commercial banks) for the bank interest rate in one regression run. However, results were perverse, in that reduced availability had a positive effect on inventories.

NICB regression study.—In a multiple regression study for the National Industrial Conference Board, Nestor Terleckyj found no evidence that the interest rate significantly affected inventory investment.[27] His study endeavored to explain changes in manufacturing and trade inventories on the basis of the ratios of new orders to sales, unfilled orders to sales, inventories to sales, recent changes in prices, and the interest rate on commercial paper. He dealt with quarterly and semiannual percentage changes in inventories after valuation adjustment over the period 1947–59. The results (the net regression coefficients) were statistically significant except in the case of the interest rate. Values calculated from the regression equation fitted actual changes in inventories very well.[28]

An advantage of Terleckyj's approach is his use of ratios and percentage changes for inventories, sales, and orders. This tends to prevent scale distortions—those due to secular growth—from creeping into the results.

The insignificant interest rate effect found by Terleckyj contrasts strongly with the Solow et al. finding, even though they used about the same time period for their analyses. There were differences in industry coverage, but this explanation is not very good. Terleckyj's use of inventory changes with valuation adjustments is a more plausible explanation, since valuation adjustments may and probably do

---

[26] The one quarter lag is technical (due to statistical requirements of identifying inventory level or change as the dependent variable). We may just as well think of the maximum change in inventory investment happending shortly after the interest rate rises or falls. In this connection, the Solow et al. statistical regression model may have indentification difficulties. Current sales and current sales changes are used as independent variables although these and inventory investment are highly interdependent.

[27] Nestor E. Terleckyj, "Measures of Inventory Conditions," National Industrial Conference Board, 1960.

[28] R^2 was .91 for semiannual changes and .79 for quarter changes in inventories. The graph of actual changes and those explained by the regression also showed very good fits in a majority of quarters.

remove an important financial demand for funds (see footnote 39 later for a discussion) when prices are rising.

An even more important difference is as follows. Terleckyj's model assumes that the impact of an interest rate change falls directly on inventory investment, and only through this does it ultimately effect the total stock of inventories desired for each given level of total sales. On the other hand, the Solow et al. structural model underlying their statistical regression postulates that changes in interest rates influence investment indirectly, but more powerfully, via a direct effect on the desired ratio of total inventories to total sales. Thus, the interest rate is allowed much more leverage in their model. Investment is so small relative to the inventory stock, even at the crest of an investment wave, that a relatively small percentage alteration of the stock will produce a larger inventory change than a relatively large change of the inventory investment flow. Finally, the Solow et al. lagged adjustment process gives an interest rate change more time in which to work itself out than the one quarter specified by Terleckyj's model.

Eisemann study.—A pioneering study by Doris Eisemann [29] found some association in manufacturing between changes in bank loans and changes in inventories. Her techniques were comparison and simple correlation of both unadjusted and seasonally adjusted bank loan and inventory data. For quarterly percentage changes in seasonally adjusted loans and inventories between 1947 and 1956, she found a rather high correlation (r=+.91) in manufacturing. Among individual manufacturing industries, these correlations were generally lower, and a few were insignificant. Finally, she found that inventory investment by small- and medium-size manufacturing corporations was associated much more with bank loan changes than was investment by large corporations. For quarterly percentage changes of inventories and bank loans between 1951 and 1956, the coefficient of correlation ranged between +.83 and +.92 for manufacturing groups under the $10 million asset size boundary. But it was only +.51 for firms in the $50 to $100 million size group, and still lower (+.35) for firms of over $100 million asset size. She also found an association between current changes in bank loans and business expectations on current inventory changes expressed 3 months earlier in Dun & Bradstreet surveys.

These findings tend to show a good if probably exaggerated [30] association between inventory investment and bank borrowing over business cycles. The suggestion of a looser association of large than small firm borrowing with inventory outlays is consistent with other evidence on structural differences in financing as between small and large corporations.[31]

A drawback is that her results do not suggest the proportion of inventory investment financed by bank loans, since she correlated percentage changes in both loans and inventories.[32] A second

[29] Doris Eisemann, "Forecasting Business Loans," Banking, July 1957, and "Manufacturers' Inventory Cycles and Monetary Policy," Journal of the American Statistical Association, September 1958. She utilized inventory data without valuation adjustment.

[30] Other investment than inventories is financed by bank borrowing, and both inventory and other investment tend to move together over the cycle.

[31] Large corporations have many alternative methods of borrowing open to them, while most small corporations depend almost exclusively on bank borrowing and trade credit for short-term finance. (Cf. "Small Business Financing: Corporate Manufacturers," Federal Reserve Bulletin, January 1961.)

[32] A technical advantage to this procedure is that if bank borrowing finances only a minor proportion of additions to inventories, correlation of percentage changes avoids scale distortions between periods when inventory accumulation is large and periods when accumulation is small. Concentration on correlation coefficients rather than regression analysis avoids the rather arbitrary choice of either bank loans or inventory changes as the "independent" variable.

drawback is her use of simple correlations. We know that business borrows for many reasons besides inventory accumulation: Extension of trade credit and payment of taxes are only two of these. A multiple regression approach would have been more appropriate.

Robinson study.—In a multiple regression study of department store inventory accumulation from 1920 to 1956, Robinson also found no interest rate effect.[33] The approach in this study was to explain monthly inventory changes by month-to-month changes in sales, prices, and the interest rate on commercial paper. The hypothesis is that inventory accumulation in any month is a function of sales changes in each of the past 5 months plus the combined change in the sixth, seventh, and eighth past months. In other words, he utilizes six independent variables representing past change in sales.

A drawback of this approach is that the interplay between total stocks and total sales is not taken into account. On the other hand, Robinson's approach has the virtue of attributing inventory decisions not only to the most recent changes in sales but also to changes somewhat earlier.

The proportion of inventory investment explained by this hypothesis was extremely low.[34] However, monthly series will usually be less highly correlated than quarterly data, since the use of quarterly data eliminates some irregular movements in both variables. Another reason for the low correlation is that department store stocks tend to be influenced heavily by special factors which are difficult to mirror in a formal regression, e.g., very severe weather, fashion changes.

Anderson study.—Anderson,[35] in his multiple regressions for manufacturing covering the 1948–58 period, attempted to explain changes in short- and long-term borrowing (his dependent variables) by inventory investment, plant and equipment outlays, changes in net accounts receivable, and other factors. He found a significant net association between inventory investment (in book value terms) and changes in short-term bank loans in manufacturing: 24 percent of inventory investment is financed by bank loans. Also, significant results were found for each asset size group. As might be expected the association between inventory investment and changes in long-term sources of funds, including term loans, was found to be much more tenuous.

This study may be criticized on two technical grounds: (1) The regression analysis took bank loans as the dependent variable; that is, it attempted to explain changes in bank loans by changes in other variables. But it assumes that bank loans change only in response to demand for loans. Changes in the supply of bank loans may also affect inventory and other investment, and the study under review throws no light on this effect. (2) If balance sheet items are being analyzed and changes in most of these items are included in multiple regression analysis, there is a tendency for any one item, taken as a dependent variable, to be highly correlated with the remaining items taken as independent variables.

Federal Reserve regression studies.—Preliminary regression studies of inventory investment, undertaken at the Federal Reserve Board, have used a lagged stock-adjustment model which is basically similar to

[33] Newton Y. Robinson, "The Acceleration Principle: Department Store Inventories, 1920–56," American Economic Review, June 1959.
[34] R^2 is 0.35 for the period 1920–36 and 0.37 for 1948–56.
[35] "A Regression Study of Manufacturing Finance: 1948–58," unpublished paper read before the Econometric Society, St. Louis, December 1960.

76626—61—pt. 2——8

that of the Solow et al. study described earlier. The relevant differences are as follows: (1) With respect to financial variables, the Solow et al. model tried alternately the cost of credit (the bank rate on short-term business loans in 19 large cities) and a credit availability variable. The Federal Reserve model tests the same interest rate plus the loan/deposit ratio of commercial banks as a credit availability variable and the liquidity ratio of corporations as an index of the availability of internal business funds to finance inventories. (2) The Solow et al. model assumes that some fraction of the current sales change is an index of expectations of sales changes currently and in the near future. For the same purpose, the Federal Reserve model tests alternate "leading indicators," e.g., the change in unfilled orders lagged one quarter, as sales predictor variables. As a result, the Federal Reserve model has only one unlagged independent variable, while the Solow et al. regression equation has two. The Federal Reserve model thus reduces, although it does not eliminate, the identification difficulties discussed earlier. (3) Unlike the Solow et al. model, the Federal Reserve model does not assume that an unforeseen change in sales is reflected, dollar for dollar, in unintended inventory accumulation or liquidation. Instead, a separate variable is introduced as an index of the impact of these unforeseen sales changes on inventories.

Except for these rather technical differences and industry coverage, the Federal Reserve structural model underlying the regression equations used is identical with that of Solow et al.

The liquidity ratio of corporations is measured by liquid assets (net of income tax liabilities) divided by current liabilities (net of income tax liabilities). This is taken as an index of the availability of internal funds to finance inventory accumulation, after the special requirements for such assets involved in changing tax liabilities is removed. The loan/deposit ratio for all commercial banks, with its 1952–60 trend removed, was taken as an index of the availability of bank credit.[36] When this ratio is high relative to its postwar trend, banks are presumably less able to lend for inventory investment and other purposes, *ceteris paribus*. The converse should be true when the ratio is low relative to its trend. The interest rate used in the regressions was the Federal Reserve series, bank rates on short-term business loans in 19 large cities.[37]

The model just described was tested on two industry groups: durable manufacturing and wholesale and retail trade (excluding auto dealers).[38] In each regression, the independent variables used were those particular to the industry, except for the interest rate and the loan/deposit ratio. A liquidity ratio variable was not used in the trade regressions, since the corporate retail and wholesale ratios computed from unpublished SEC industry data were thought not to be representative of noncorporate trade liquidity.

---

[36] The trend was removed because of a hypothesis that bank attitudes toward proper levels of this ratio were changing during the 1950's. Removal of trend also reduces the correlation between the ratio and the interest rate to a very low level.

[37] This was selected rather than a more volatile rate, i.e., that on commercial paper of 4 to 6 months' maturity, because (*a*) it measures rates paid by most borrowers financing inventory investment. (*b*) a variable tending to reflect only the cost of money is more suitable for use in conjunction with other independent variables purporting to measure credit availability, and (*c*) it avoids some volatility of commercial paper rates due to special movements in the Government securities markets. In one respect, the bank loan rate is not sensitive to changes in the cost of bank loans. It does not reflect rises in compensating balance requirements over recent business upswings and thus understates cyclical changes in effective interest rates (since interest is also charged on the compensating balance held back from a loan).

[38] Auto dealers were excluded because changes in their inventories tend to dominate total retail inventories, while their borrowing is thought to be rather insensitive to changes in credit conditions.

Some equations were tested both with and without inventory valuation adjustment, as an indirect test of the effect of changing prices on the dollar volume of inventory needs.[39]

On the whole, results were disappointing. They are published primarily as information to research workers in the same field. However, some tentative conclusions may be of more general interest.[39a]

(1) In the case of durable manufacturing, interest rates were found to have an uncertain influence on inventory accumulation. In three regressions involving different sales predictor variables, the partial regression coefficient for the interest rate was negative each time, but it was never statistically significant at the 5 percent level. However, all three coefficients were close to their standard errors in size.[40] In trade, the interest rate coefficients were perverse in sign (positive) until the loan/deposit ratio was included as a variable. However, see (2) and (3) on this point.

As a test, the Solow et al model was tried on the same data and the same financial variables. The sign and magnitude of the interest rate coefficients in durable manufacturing were much the same in both models. The Solow et al model produced a sizeable and statistically significant interest rate coefficient for trade, but only after putting in the loan/deposit ratio as an additional financial variable.[41] And the positive value of the coefficient for the latter (see below) deprives the interest rate finding of value.

(2) The results for the corporate liquidity variable were not statistically significant.[42] In each model tried, the standard error of its net regression coefficient was several times larger than the coefficient itself, which turned out to be very small.

(3) The statistical results for the loan/deposit ratio for commercial banks were perverse, in that the loan/deposit ratio was associated positively with inventory investment. (In most cases, this result was statistically significant.) Literally this means that reductions in the ability of banks to expand loans, as measured by the loan/deposit ratio, produce *higher* rates of inventory investment, and vice versa. One interpretation would be that demand forces have been dominant in the market for business loans, and banks have supplied customers with most of what they need. (In this case, the assumptions of the Anderson study would be broadly correct.) Or, credit availability variables (including the loan/deposit ratio) may contribute to an investment explanation only during periods of credit restraint.

(4) In explaining inventory investment for the 1952–61 period these regression analyses were about as successful as those of Solow et al. for the same period, both for durable manufacturing and for trade. For durable manufacturing, the proportion of inventory investment explained by all the variables together (R^2) ranged between 75 and 79 percent. For wholesale and nonauto dealer retail trade,

[39] For present purposes, failure to adjust inventory investment for price changes implies that businesses respond similarly to increased (or decreased) financial requirements whether these stem from a change in volume or a change in price of inventories. On the other hand, an inventory valuation adjustment implies that price changes have no effect on the external finance required. In all probability, the truth lies somewhere between these two extreme assumptions. But since Federal income taxes are paid on inventory (book) as well as other profits, it probably doesn't lie too near the IVA assumption.

[39a] A memorandum describing these results in detail is available on request.

[40] If a net regression coefficient equals its standard error, and if the distribution of such coefficient estimates is the normal one, there are 83 chances out of 100 that the true coefficient in the universe is of the same sign (negative in this case).

[41] However, the sign of the regression coefficient for this variable was perverse (a plus). See below.

[42] Federal income tax liabilities were deducted from both assets and liabilities on the grounds that this procedure is better than the alternative of ignoring the need to set up liquidity reserves against this particular liability.

the corresponding proportions explained were much lower, between 56 and 59 percent.[43] The poorer results for trade may come partly from the lower reliability of inventory and sales data in this area as compared with manufacturing. Furthermore, irregular influences like weather and changes in fashion are also more important for trade than for manufacturing inventories.

(5) A central problem in this and other regression studies is to express how businessmen form expectations of future sales, since these are an important determinant of production and materials procurement schedules and intended inventory accumulation. All but one study consulted [44] have assumed current sales expectations to be a simple linear function of output, sales, new orders, capacity, etc., one or two quarters earlier. But the more distant past surely has some weight in sales expectations.

This observation bears on the question of credit cost and availability effects. Over the postwar period, changes in the cost and availability of credit have tended to favor inventory accumulation in the early stages of a recovery and to restrict it in the late stages. At the same time, if the recent change in sales is taken as an indicator of expected changes in sales and therefore as a determinant of inventory policy, there would be greater inventory investment during the earliest stage of recoveries and less later. In fact, however, inventory investment in manufacturing has tended to be low or negative early in the upswing and to be relatively high late in the upswing. From this, either informal analysis or multiple regression studies using data of the recent past would tend to conclude that the cost and availability of credit had virtually no effect on inventory outlays.

It is possible, however, that cumulated expectations explain why inventory investment tends to build up in the later stages of a recovery, in the face of increasing credit tightness. When a recovery is in its early stage, expectations of sales increases are uncertain because the past data relevant to expectations include recession quarters. After the recovery has progressed for a while, however, expectations become more optimistic and/or less uncertain as recession experiences recede from memory horizons.[45] This behavior of expectations is consistent with available data on differences between expected and actual sales of manufacturers, as the following table shows:

[43] It makes quite a difference to the multiple correlation coefficient whether we take the level of, or changes in, a series as the dependent variable. For example, R^2 jumped from 0.66 to 0.99 in the equation for wholesale and nonauto retail trade, when the level of inventories was substituted for changes in inventories (investment) as the dependent variable.

[44] The exception is an unpublished study by Peter Pashigian of Northwestern University, "Business Expectations and Inventory Investment."

[45] Cumulated expectations, for example, would explain why fears of a deep depression persisted into the early 1950's in the face of 3 years of rapid expansion and one moderate depression (1949). The theory of expectations based only on the recent past would not explain the persistence.

Anticipated and actual percentage changes in sales, manufacturing (year-to-year changes) [1]

Year	Anticipated	Actual	Anticipated-actual
1955	+4	+12	−8
1956	+6	+5	+1
1957	+8	+3	+5
1958	−2	−9	+7
1959	+9	+13	−4
1960	+8	+3	+5

[1] Anticipations are collected by the SEC and the Commerce Department in January or February of the year to which the change refers. The percentage base is sales of the previous year.

Therefore regressions studies using cumulated past data as expectational variables might yield different results on the association between credit cost and availability and inventory movements.

Other regression studies.—Multiple regression explanations of inventory investment by Darling,[46] Duesenberry-Eckstein-Fromm,[47] and Lovell [48] should be mentioned. Although none of the published accounts of these studies states whether a credit cost (or availability) variable was tried, the inference exists that if tried, it would not have explained a significant portion of investment.[49]

United Kingdom survey, 1958.—In March 1958, the Association of British Chambers of Commerce (A.B.C.C.), gathering evidence for the Radcliffe Commission, asked its member chambers to send questionnaires to 16,000 companies dealing with effects of the 7 percent bank rate and other credit restrictions imposed after the Suez crisis. Replies from 3,404 companies were analyzed, an effective response rate of 21 percent. These responses were tabulated by size of firm and by industry but not cross-classified by both. The questions and answers directly relevant to the present paper are given in table 1.[50]

[46] Paul Darling, "Manufacturer Inventory Investment, 1947–58," American Economic Review, December 1959.

[47] Duesenberry, Eckstein, and Fromm, "A Simulation of the U.S. Economy in Recession," Econometrica, October 1960.

[48] Michael Lovell, "Manufacturers' Inventories, Sales Expectations, and the Acceleration Principle," Econometrica, July 1961.

[49] Lack of autocorrelation in the residuals, as shown by the standard tests used, is at least presumptive evidence that additional variables, such as the cost of bank credit, will not add to the explanation of inventory investment. Furthermore, explaining a high proportion of investment by nonfinancial variables implies at least that the proportion explained by financial factors is low.

[50] For an analysis of this survey, see W. H. White, "Bank Rate Vindicated? Evidence before the Radcliffe Committee," The Bankers' Magazine (London), August 1959.

TABLE 1.—*Selected replies to chamber of commerce survey*

	Percent of all respondents			Percent answering "Yes" in each employee size class			
	Yes	No	Not answering	Under 100	100–250	250–1,000	Over 1,000
II. Have you, since September 1957, experienced—							
(*a*) A reduction (other than seasonal) in turnover	53	46	1	58	51	50	39
(*b*) A reduction (other than seasonal) in your stocks and/or work in progress	37	60	3	43	37	31	21
(*c*) Reduced fixed investment projects	24	59	17	24	25	25	23
(*d*) Postponed fixed investment projects	32	56	12	31	34	34	30
(Total number of respondents)				(1,679)	(725)	(685)	(315)
	Percent of all respondents checking answer			Percent checking answer in each employee size class			
				Under 100	100–250	250–1,000	Over 1,000
III. The principal reason for the reduction in turnover, the reduction in stocks, and/or the reduction or postponement in fixed investment projects was: [1]							
1. A general slackening of business			30	33	30	27	21
2. Tightness of money among your customers			11	13	10	8	8
3. The increased cost of borrowing			4	3	5	5	4
4. Greater difficulty in obtaining finance			5	5	6	6	5
5. Hire-purchase restrictions			1	}			
6. Increased competition			2	}			
7. Altered assessment of trading prospects			3	} 8	9	11	17
8. Cuts in Government and nationalized industry investment programs			3	}			
9. Any other cause			2	}			
10. Not answering question III			40	38	40	42	44

[1] The original questionnaire asked respondents to rank the importance of all 9 factors subsumed under III but the published report does not tabulate 2d and 3d ranking factors. Only the 1st-ranking factor is shown here and the question is modified accordingly.

Source: Committee on Working of the Monetary System, vol. 2, pp. 85–96.

Overlooking for the moment the problem of low response, we may note that this survey tends to show a higher responsiveness of investment to credit conditions than was indicated by such earlier surveys as those of Meade and Andrews in the late 1930's. Four percent of all respondents listed the increased cost of borrowing as the principal reason for reduced sales and/or investment during the 1957 credit squeeze. An additional 5 percent listed decreased availability of credit of all kinds (including capital issues) as a principal reason, and an additional 11 percent listed tightness of money among customers. Thus 20 percent of all respondents listed credit factors as the chief determinant of their reduced sales and/or investment. Seventeen percent of all large respondents (those with more than 1,000 employees) answered in this way.

Among those firms that had experienced a reduction in sales or had reduced fixed investment or inventories (60 percent of all respondents), 7 percent attributed it to higher interest costs and 33 percent to one of the three credit factors (III 2, 3, and 4 in table 1). Even these latter percentages exclude firms which would have answered that sales and/or investment did not rise as much as they would have in the absence of the 1957 credit restraint policies. With respect to inventories in particular, the question is very restrictive; it asks only whether stocks were reduced. Thus firms were not expected to answer "Yes" if they added to stocks at all, even if the additions were much smaller than they would have been in the absence of credit restraint. In the case of fixed capital outlays, they were asked only if spending was reduced or postponed.

On the other hand, the tabulated answers include credit restraint effects on sales and fixed capital investment as well as on inventories. Moreover, it is not possible to compare numbers of firms experiencing reductions in stocks with those reducing or postponing fixed capital outlays since it is not evident how many checked one or both of questions II(*c*) and II(*d*).

An additional indication of responsiveness to credit cost is that 30 percent of all respondents took "steps to reduce or pay off overdraft because of increased costs." Twenty-eight percent of large firms answered this way. At least 13 percent of all respondents had their overdrafts reduced, or were asked directly to reduce or pay off overdrafts by their banks. (The form of the question precludes measuring the percentage of firms affected by *cost* and availability but it was certainly over the 30 percent affected by cost.)

The low response rate in this survey (21 percent) raises the question whether the results exaggerate the effects of credit restraint since firms whose operations were affected may have been more likely to take the trouble to fill out a questionnaire than firms that were relatively unaffected. The Radcliffe Committee believed this and minimized the survey findings for this reason. However, some businesses which had been affected by credit restraint may not have responded, or may have responded incorrectly, from motives related to self-esteem. Success at the bank is at least as much an index of overall business success as, say, good labor relations. Furthermore the larger is the true proportion of firms not affected by credit restraint to firms so affected, the more such motives could bias a survey toward underreporting of financial difficulties or offset nonresponse among firms with no such difficulties.[51] In short, judgments on the proclivities of nonrespondents are hazardous.

In any case, the response rate for firms of over 1,000 employees, which account for a large share of British inventory investment,[52] was undoubtedly much higher than the overall survey rate of 21 percent. White concluded that there cannot be many more than 1,000 firms of this size in the British economy, and independent estimates support his conclusions.[53] From this aggregate, those not members of cham-

[51] If the true proportion of firms unaffected by credit conditions to firms so affected is a/b, the respective proportion among nonrespondents must be greater than a/b before the survey understates this proportion. For example, there would have to be more than four firms unaffected and not responding for each firm affected and not responding, before a survey would overstate a true proportion of 20 percent of all firms affected by credit conditions. And this does not take account of self-esteem biases among respondents, which may more than offset a relative preponderance of nonrespondents not affected by credit conditions.

[52] In 1954, such firms employed 31 percent of all workers, in British manufacturing. Allowing for such factors as proportions of workers to supervisory and staff personnel, and structural differences between manufacturing and other industries, this still implies that firms of the size cited normally undertake one-fourth or more of business inventory investment.

[53] White, "Inventory Investment," page 173. The 1954 Census of Production for the United Kingdom reported 1,054 manufacturing establishments with over 1,000 employees each. Extrapolation to the survey data by 1951–54 census relationships yields an estimate of just about 1,100 such establishments. Establishments are not firms, and we should add to the manufacturing total establishments outside of manufacturing. Estimating the latter by the proportion of nonmanufacturing to manufacturing respondents of all sizes in the A.B.C.C. survey, we would have 1,406 establishments in all industries of the specified employee size class. Response rate factors associated with industry and size undoubtedly make this estimate too high; the true number of such establishments is probably between 1,150 and 1,300. Taking into account differences between establishments and firms, there would not have been significantly over, say, 1,100 firms in the United Kingdom of the size class mentioned, in mid-1957. (More units fall out because many large establishments are owned by few large firms than are added because aggregating workers of multiestablishment firms moves the latter into the 1,000 and over employee size class.)

bers of commerce should be deducted in evaluating nonresponse, since nonmembers were not sent questionnaires. Finally, the member chambers were asked by the association to send questionnaires to "representative samples" of 30 percent of their members. This loose reliance on judgmental sampling probably resulted in circularizing over 30 percent of large members. But as mentioned before, the relevant universe for evaluating nonresponse is large firm members of chambers of commerce.[54] Even if all firms belonged to member chambers of the association, and even if as many as 60 percent of these were sent questionnaires, the large firms' response rate would still be about 48 percent using the estimates of 1,100 large firms in Great Britain. More plausible assumptions in this chain of reasoning would lead to a response rate between 70 and 90 percent for large firms.

The response rate for firms with from 250 to 1,000 employees each may have been low. But from the answers given in table 1, we can at least infer a strong probability that if the large firm sample was not seriously biased toward overreporting of credit and/or credit cost difficulties, neither was the sample of medium-large firms (with 250 to 1,000 employees each). Nineteen percent of medium-large firms checked III, 2 and/or III, 3 and/or III, 5 (the questions dealing with credit conditions), compared with 17 percent of the large firms. But 27 percent of the former checked "a general slackening of business," compared with 20 percent of the large firms. Finally, the percentages not checking any of question III were almost identical between the two size groups; and question III specifically implies that they couldn't pass this by unless they had checked no to all of question II. In other words, while medium-large and large firms had about the same experience (with respect to percentages experiencing reductions in investment, fixed investment, and/or inventories and/or postponement of inventories), relatively more medium-large firms checked a noncredit reason (slackening of business) as the primary reason for the reductions and/or postponements referred to. This would scarcely support a serious overweighing of the medium-large firm sample with firms having credit difficulties, if the large firm sample is not seriously biased in this respect. And while nonresponse to II and III blur this conclusion, biases of this type could be in either direction.[55]

For present purposes, the most serious gap in the survey is the failure to differentiate among credit restraint effects on sales, inventories and fixed capital outlays. However, 37 percent of all respondents indicated a reduction of inventories after September 1957. In view of the extreme mildness of the decline in output in the United Kingdom after September 1957, there is a presumption that a substantial share of firms checking credit cost and availability in question III had inventories in mind as well as fixed capital outlays and sales. It would appear therefore that this survey tends to show a significant impact of credit restraint on inventory investment in the United Kingdom in 1957–58.

United Kingdom survey, 1952–53, 1954–55, 1956–57.—Another British survey, also for the Radcliffe Commission, was undertaken by

[54] There are good reasons to believe that a substantial proportion of large firms do not belong to the member chambers of the association.

[55] It might be the case that medium-large firms had suffered much greater reductions in sales after September 1957 than large firms had. In this case, the low medium-large firm percentages checking credit conditions and the high percentage checking "slackening of business" could be explained away. But the slightness of the output decline after September 1957 in the United Kingdom does not suggest this type of differential cyclical impact by size of firm.

the Federation of British Industries (F.B.I.) in the summer of 1957.[56] A mail questionnaire was sent to all members, numbering between 5,000 and 6,000 manufacturing companies (excluding operating subsidiaries that do not make financial decisions). A total of 1,595, or between one-fourth and one-third of the firms responded. While the chamber's survey covered the 7 months after September 1957, the federation survey asked manufacturers about their reactions to credit conditions in their financial years, 1952–53, 1954–55 and 1956 to mid-1957. Thus the survey does not cover the period of severe credit squeeze after September 1957, when bank rate was at 7 percent.

A defect of the federation survey is that it failed to differentiate explicitly, as did the chamber of commerce survey, between credit cost and availability effects on investment. On the other hand, the federation survey asked specifically about the effects of credit conditions on inventory outlays. Also, the federation survey traces changes in the impact of credit conditions over 6 years.

The pertinent results are summarized in table 2. It shows percentages of respondents indicating a positive effect of interest rates on borrowing and the effect of cost and availability of credit on inventory and fixed capital outlays from 1952 through 1957.

An outstanding feature of the results is the increasing sensitivity to credit conditions from 1952–53 to 1956–57. In the early years, only 4 percent of the respondents reduced stocks because of the cost or difficulty of borrowing. In 1956–57, 11 percent reduced stocks for this reason. With one minor exception, the number of firms indicating some response to credit difficulties doubled or more than doubled from 1952–53 to 1956–57. During this period interest rates were rising and internal liquidity of firms was declining.

In the 1956–mid-1957 period, 11 percent of all respondents indicated a reduction of stocks, and 12 percent a decision against raising stocks, as a result of difficulties in raising external funds. These two percentages cannot be added because an unknown number of respondents checked both answers. However, neither percentage includes firms which did increase investment but not by as much as they would have in the absence of credit restraint.

[56] Committee on Working of Monetary System, Memoranda, vol. 2, pp. 118–122.

TABLE 2.—*Selected replies in the Federation of British Industries Survey (percent of all respondents)*

Question	Employee size class (number of employees per firm)	Percent answering yes,[1] 2 years ending in— 1953	1955	1957
During any of the periods indicated were you deterred or prevented from raising extra money because of: (a) The cost of borrowing [1]	All firms (I)	2	3	6
	Up to 200 (II)	4	5	8
	201 to 700 (III)	2	4	7
	Over 700 (IV)	1	1	4
In any of the periods indicated, did the difficulty of raising extra money outside the firm (including both the cost of borrowing and the administrative difficulties involved) cause you to: (a) Reduce your stocks	I	4	6	11
	II	5	8	12
	III	4	7	12
	IV	3	4	10
(b) Decide against raising your stocks	I	4	8	12
	II	7	12	17
	III	3	7	9
	IV	2	4	8
(c) Decide to postpone fixed investment which you would otherwise have undertaken.	I	5	9	16
	II	8	12	15
	III	4	9	17
	IV	4	7	17
Percent of those answering yes to any question (including d, e, and f).[2]	I	10	17	27
	II	12	22	31
	III	9	17	25
	IV	8	12	24

[1] (b), (c), and (d) comprise rejection of issues by the Capital Issues Committee and "any other reason."
[2] These referred to reducing "trading," not raising "trading," and any other effects, respectively.

NOTE.—The total number of respondents by employee size class was:

Under 200	593
201 to 700	447
Over 700	555

Source: Committee on Working of the Monetary System, vol. 2, pp. 119–122.

The percentages of firms indicating that interest cost was a cost deterrent to borrowing (6 percent in 1956–57) appears low compared with the percentages stating that difficulties of raising funds had adverse effects on investment and other operations (27 percent). This gap can be traced to credit availability effects. In another question, firms which had had overdraft facilities prior to the survey date (mid-1957) were asked whether in 1955, or 1956, or early 1957, their banks had met "their requirements in full" or had not (required a reduction of overdraft limits, required a reduction of overdrafts, refused an increase of overdrafts, and/or had not increased overdrafts as much as respondent wanted).[57]

Unfortunately, the answers were not tabulated for all firms answering "yes" to any of these or for the different possible combinations of the five questions which could check with more than one answer. We cannot therefore find the total number of respondents who had not had their requirements met "in full." But in early 1957, 5.6 percent of all respondents had been requested by their banks to reduce their overdrafts; 4.9 percent had been allowed to increase overdrafts but by less than they wanted; and 5.9 percent had not been allowed to increase overdrafts. While some overlap of firms among these three answers is logically possible, because of the half year of experience covered by the question, it would be surprising if less than 10 percent of all respondents had checked one or more of these three answers. This

[57] Ibid., p. 120, vol. 2.

would account for half of the discrepancy referred to above. The remaining one-half of the discrepancy can probably be explained by (*a*) firms having difficulties with nonbank sources of finance and (*b*) firms not making formal attempts to raise funds because they thought the efforts would not be successful.[58]

As table 2 indicates, inventory investment was less affected by credit conditions in the case of larger than of smaller firms. But the differential impact narrowed between 1952 and 1957.

In replying to a question on the effect of one specific change in credit cost (a rise in bank rate from 3 to 4.5 percent in January and February 1955), more businessmen checked "reduce or defer raising your stocks of purchased goods because of its effect on costs" than checked "defer an investment decision or substantially reduce an investment project because of its effect on costs." The cost factor was apparently at least as important for inventories as for fixed capital.[59]

Response rates among large firms were probably not as good as in the A.B.C.C. survey. This is partly because "large" is defined in the F.B.I. survey to include firms with over 700 employees, and experience with both United Kingdom and United States business surveys has shown a positive correlation between size and degree of response. From the 1951 and 1954 United Kingdom Censuses of Production, we can estimate the mid-1957 number of manufacturing establishments in this employee size class to have been about 1,800. (The United Kingdom censuses use employee size classes, as did the F.B.I. survey, but differences in class limits and the need to extrapolate to mid-1957 necessitate some estimation.) Making allowance for probable differences between the firm and establishment populations of the United Kingdom, there would probably be between 1,300 and 1,600 manufacturing firms with over 700 employees. Some of these would not be members of the federation. But since the federation sent questionnaires to all of its members, the large firm response of 555 questionnaires implies, at the most, a response rate of about 50 percent (estimating 1,000 F.B.I. member firms in this size class). The most probable response rate for this size group would be in the neighborhood of 40 percent.

However, one factor accounting for this low response would appear to be random in nature, hence not producing a bias in the sample.

---

[58] In the Federal Reserve survey of manufacturing finance (cf. Federal Reserve Bulletin, January 1961), it was found that 4 out of each 11 firms needing short-term credit and "dissatisfied with their financial experience" made no efforts to obtain credit. An additional reason for the discrepancy discussed may have been respondent interpretation of the first question (table 2) in terms of only long-term borrowing.

[59] The special tabulations for the same question also provide some insight into the characteristics of firms sensitive to credit restraint. A total of 175 firms, or 11 percent of all respondents, said that the January–February 1955 rise in the bank rate was a major factor in taking business decisions. These firms were scattered over all types of industries and all size groups. The rate of return expected on new investment was not significantly different for these firms especially sensitive to bank rate than for all of the respondents, as can be seen in the following table:

Gross rate of return [a] expected on new investment	Percent of firms indicating some credit effects	Percent of all respondents
0 to 10 percent	6	7
11 to 20 percent	53	45
21 to 30 percent	20	25
31 to 40 percent	8	6
Over 40 percent	0	1
No answer	13	16

[a] Probably before depreciation, although the survey did not define this term.

The questionnaires were sent to F.B.I. members during the summer of 1957, when many board chairmen were on vacation. If we exclude these firms as having been, in effect, not drawn in a random sample, the "true" response rate could be well over 50 percent for the "over 700 employee" size class.

Unless the vacation factor explains much nonresponse, the response problem is still probably greater for the F.B.I. than for the A.B.C.C. survey, with respect to aggregate inventory investment. The 24 percent of large firms indicating some effect of 1956–57 external financing difficulties on investment and/or operations, in the F.B.I. survey, is not unbelievably high. But it looks high compared with the 8 to 17 percent of large firms in the A.B.C.C. survey which indicated direct or indirect credit cost and availability effects on investment or operations. Although the A.B.C.C survey covered a shorter period (about 5 months), the latter period saw much higher cost and lower availability of credit, than the 1956–mid-1957 one. Intuitively, we would expect more response of some kind to credit conditions in the later than during the earlier period. While this is no reason whatsoever to reject the FBI findings, they should, for the reasons enumerated, be taken with some caution.

Oxford survey of 1938.—This survey should be touched upon because of extensive references to it in economic literature. It consisted of detailed questions put to 37 businessmen and covering credit cost and availability effects on investment of all types. The results have been widely interpreted as a negative finding, since—

> there is almost universal agreement [among businessmen questioned] that short-term rates of interest do not directly affect investment either in stocks or fixed capital.[60]

The reliability of a sample of 37 respondents for all industries in Great Britain is somewhat obscure. All but two were executives "mainly representing prosperous firms in a strong financial position." [61] The survey was conducted in 1938, a recession year in Great Britain as well as the United States; conditions typical of a short recession were added to the lingering effects of the great depression. Demand for credit was thus weak and the supply abundant in general, adding to the normal relative insulation of "prosperous firms" from credit supply factors.

It is true that respondents were asked whether they had ever felt some effects of credit cost and availability. However, depression conditions had prevailed since 1929, and investment demands in Great Britain were not high even during the 1920's. Furthermore the individual replies to the questionnaire can be interpreted quite differently from the "almost universal agreement" cited by Meade and Andrews. In conclusion, this survey offers little applicable to current conditions, and the validity of the Meade and Andrews conclusion for the prewar period may be questioned.

Oxford Economic Institute Survey, 1939.—This survey consisted of mail questionnaires to 1,000 manufacturing firms (selected by simple

[60] J. E. Meade and P. W. S. Andrews, "Summary of Replies to Questions on Effects of Interest Rates," Oxford Economic Papers, October 1938, p. 28.

[61] J. E. Meade and P. W. S. Andrews, "A Further Inquiry Into the Effects of Rates of Interest," Oxford Economic Papers, February 1940, p. 33.

random sampling) plus 308 "public companies," i.e., those listed on stock exchanges, which are nearly all large firms in Great Britain.[62] Response rates were 25 percent for the 1,000 manufacturing firms and 19 percent for the 308 "public companies."

Unlike the earlier survey of 37 large, prosperous companies, this one uncovered some effect of credit cost and availability on investment. Fifteen percent of respondents checked "yes" to the question, "Have interest rates, the yield on government securities, and/or the facility with which you can raise new capital from the public ever affected the size of your holdings of stocks?" Twenty-five percent affirmed that the size of stocks, repair and maintenance expenditures, and/or decisions on plant extensions had been affected by one or more of these factors.

As in the postwar British surveys, interpretation of these sample findings is obscured by the high degree of nonresponse.[63] The year 1939 was of generally high business liquidity and low demand for funds, since the British economy was still climbing out of the great depression. The questionnaire did ask businessmen whether they had ever been influenced in their investment by credit cost and availability. However, depression conditions had persisted for 10 years and recollections of earlier periods were undoubtedly hazy.

Concluding comments

This study does not presume to have reached definitive conclusions on the effects of changing credit conditions on inventory accumulation. Nevertheless, the following propositions appear to be reasonable.

Both *a priori* considerations and the empirical evidence reviewed above suggest that the direction of the effect is such that when tighter credit prevails, inventory investment tends to be deterred. When credit conditions are easier, this deterrence weakens or becomes negligible.

The magnitude of the effect, while probably not negligible, remains in doubt. But there are theoretical and statistical reasons to suppose that regression studies tend to understate it under present-day conditions. All studies of inventory investment are hampered by the fact that the available statistics show the net results of both intended and unintended inventory investment [or disinvestment], while regression and other explanations of underlying business behavior deal mainly with intended investment, which is not directly observable. This does not necessarily result in understating the influence of financial more than that of nonfinancial variables. However, there remains the statistical difficulty of separating the effect of credit conditions from the effect of the other determinants of inventory investment, which themselves may be influenced by credit conditions. Also, regression studies have not been successful in measuring credit availability, which tends to reinforce the [measurable] cost of credit. The result is that regression studies probably understate the relationship between credit conditions and inventory movements.

[62] Ibid.

[63] For a critique of this survey, see W. H. White, "Interest Inelasticity of Investment Demand," American Economic Review, September 1956. White infers that there was no serious response bias towards overstating credit effects in this survey.

The survey results tend to show strong effects of credit conditions but raise questions not only because of the response problem but also because British firms may be more sensitive to changes in credit cost and availability than their American counterparts. Furthermore, these changes have been of greater amplitude in Britain, and have started at higher initial levels.

Finally, recent statistical studies include data for earlier postwar years when both business and bank liquidity were higher and inventory investment was probably less dependent on external financing than currently. In those same years, "real" interest rates rose much less than nominal rates and availability effects may have been concentrated on bank credit other than loans to business. It is possible therefore that currently, as well as in the future, credit conditions may have a greater influence on inventory investment.

BIBLIOGRAPHY

R. G. Hawtrey, Capital and Employment, 2d ed., 1952, New York, Longman, Green & Co.

P. W. S. Andrews, "A Further Inquiry Into the Effects of Rates of Interest," Oxford Economic Papers No. 3, February 1940.

J. E. Meade and P. W. S. Andrews, "Summary of Replies to Questions on Effects of Interest Rate," Oxford Economic Papers No. 1, October 1938.

R. S. Sayers, "Business Men and the Terms of Borrowing," Oxford Economic Papers, February1940.

Michael Lovell, "Manufacturers Inventories, Sales Expectations, and the Acceleration Principle," Econometrica, July 1961.

Robert Solow, E. Cary Brown, Albert Ando, and John Karaken, "Lags in Fiscal and Monetary Policy," unpublished discussion paper for the Commission on Money and Credit.

Peter Pashigian, Northwestern University, "Sales Anticipations as a Determinant of Inventory Investment," mimeographed paper.

Paul Darling, "Manufacturers Inventory Investment, 1947–58," American Economic Review, December 1959.

James Dusenberry, Otto Eckstein, and Gary Fromm, "A Simulation of the U.S. Economy in Recession," Econometrica, October 1960.

W. H. White, "Interest Inelasticity of Investment Demand," American Economic Review, September 1956.

W. H. White, "Inventory Investment and the Rate of Interest," Banca Nazionale del Lavoro Quarterly Review, June 1961.

Lloyd Metzler, "Factors Governing the Length of Inventory Cycles," Review of Economics and Statistics, February 1947.

Franco Modigliani, "Business Reasons for Holding Inventories and Their Macro-Economic Implications," Problems of Capital Formation (NBER Studies in Income and Wealth, vol. 19).

Ruth Mack, "Characteristics of Inventory Investment: The Aggregate and Its Parts," Problems of Capital Formation (NBER Studies in Income and Wealth, vol. 19).

Committee on Workings of Monetary System (Radcliffe Committee): Memoranda and Hearings.

T. M. Whitin, Theory of Inventory Management, Princeton, 1953.

Thomas Mayer, "Plant and Equipment Lead Times," Journal of Business of the University of Chicago, April 1960.

Moses Abramovitz, "Inventories and Business Cycles," National Bureau of Economic Research, 1950, New York.

Lloyd Metzler, "The Nature and Stability of Inventory Cycles," Review of Economics and Statistics, August 1941.

"Statistics of Business Inventories" (Report of Consultant Committee on Inventory Statistics, Board of Governors of the Federal Reserve System), November 1955.

Doris Eisemann, "Manufacturers' Inventory Cycles and Monetary Policy," Journal of the American Statistical Association, September 1958.
W. H. Locke Anderson, "A Regression Study of Manufacturing Finance," unpublished paper presented at the Econometric Society meeting in St. Louis, December 1960.
Nestor E. Terleckyj, "Measures of Inventory Conditions," Technical Paper No. 8, National Industrial Conference Board, 1960.
Newton Y. Robinson, "The Acceleration Principle: Department Store Inventories, 1920–56," American Economic Review, June 1959.
W. H. White, "Bank Rate Vindicated? Evidence Before the Radcliffe Committee," The Bankers Magazine, London, August 1959.
"Financing Small Business" (Report to the Committees on Banking and Currency and the Select Committees on Small Business, U.S. Congress, by the Federal Reserve System), pts. I and II, April 1958.

FACTORS DETERMINING MANUFACTURING INVENTORY INVESTMENT

BY

MICHAEL C. LOVELL

The Cowles Foundation for Research in Economics
Yale University

76626 O—61—pt. II——9

FACTORS DETERMINING MANUFACTURING INVENTORY INVESTMENT [1]

PART I. INTRODUCTION AND SUMMARY

Fluctuations in manufacturers' inventories, particularly stocks of durable goods, have played a major role in the generation of postwar business recessions. During periods of defense mobilization, inventory accumulation has been a major component of effective demand. This paper reports on an econometric investigation continued over a number of years of the influence of changes in sales volume, unfilled orders, Department of Defense procurement, and other factors affecting the level of inventories held by manufacturing firms. The conclusions of the investigation may be summarized succinctly, but only at the risk of inadequate qualification. In the summary that follows reference is provided to sections in the body of the study paper where the underlying analysis is to be found.

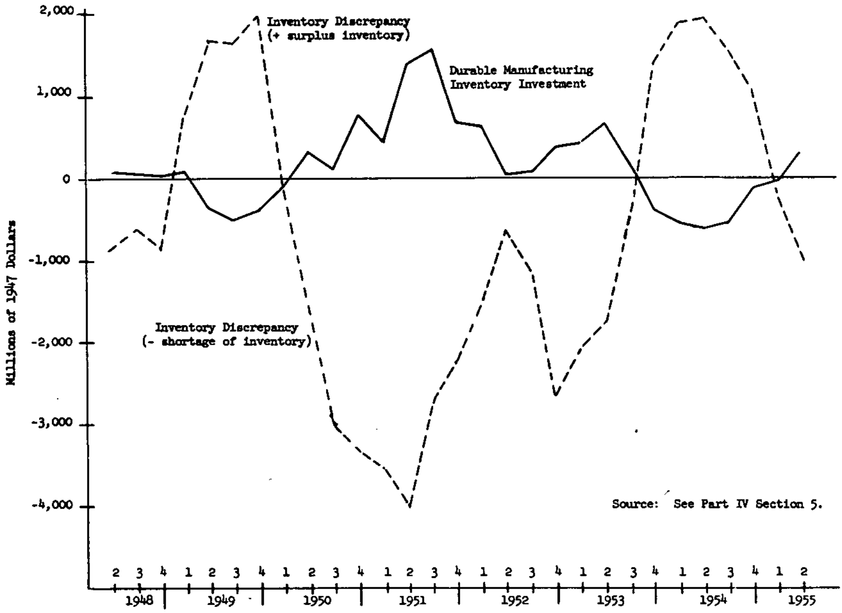
1. *A flexible accelerator buffer-stock model provides an appropriate framework for the analysis of the behavior of manufacturers' inventories.*—The details of the theoretical model utilized in the empirical investigations are elucidated in part II of the paper. Within the context of this model the behavior of inventories is explained by sales volume, unfilled orders, and errors in anticipating future sales. The major part of the empirical investigation based upon this theoretical framework was completed some time ago and is reprinted from *Econometrica* as part IV of this study paper. Deflated quarterly data for individual durable goods industries through 1955 were available for that study. It was also possible to work with deflated time series for durable and nondurable inventory aggregates classified by stage of fabrication; finished goods inventories could be analyzed apart from purchased materials and goods in process. More recently a second set of computations bringing the earlier study up to date has been completed. This second set of computations, based on deflated durable and nondurable data extending through 1960, constitutes part III of this report. In this more recent study it was necessary to work with total inventories rather than explain separately the behavior of finished goods inventory as opposed to stocks of purchased materials and goods in process; it was not possible to conduct the investigation on an industry-by-industry level of aggregation.

2. *Durables manufacturing firms tolerated a considerable shortage of inventories during the Korean war.*—In the first year of the Korean mobilization durables manufacturing firms accumulated some $2Z billion of inventory, measured in terms of 1947 prices, as may be seen on graph 1. This dramatic rate of inventory investment was not sufficient to prevent a considerable deficiency of stocks from developing during this crucial period of economic expansion. Inventories

[1] Computer and research time for this paper were provided through the generosity of the Cowles Foundation for Research in Economics at Yale University. I am indebted to Frederick W. Deming, Karen H. Hester, Donald Hester, Bruce Morgan, and Ronald Soligo for valuable suggestions and assistance with the computations.

remained below the level warranted on the basis of sales volume and the accumulation of unfilled orders. A crude estimate of the discrepancy between equilibrium and actual inventories in durable manufacturing is presented on the graph.

Graph 1. Inventory Investment and Surplus Inventories, 1948–1955, Durables Manufacturing



The details of the procedure by which this estimate was derived are presented in part IV, section 5 below. Inventory investment might well have been considerably greater during the Korean period if it were not for two characteristics of current inventory practice. First of all, firms in manufacturing follow a flexible inventory policy, attempting only a partial adjustment of actual inventories to their equilibrium level during each production period. Second, there seems to be a slight tendency for firms, on the average, to underestimate actual changes in sales. It is tempting to hypothesize that during the Korean crisis these two factors, by reducing actual inventory investment, served to limit effective demand and consequent inflationary pressure. It must also be mentioned, on the other hand, that if the marginal desired inventory coefficients relating equilibrium inventory to sales had been smaller the inventory buildup and consequant inflationary pressure might well have been reduced.[2]

3. *Department of Defense procurement activities have an immediate impact upon durable goods inventories in advance of actual expenditure.*— The level of current spending is not an adequate index of the impact of procurement upon the economy. As soon as obligations (new orders) are let to private firms there is a tendency for inventories to

[2] The implications for the stability of the economy of changes in the parameters of the equation describing buffer-stock inventory behavior have been examined at the theoretical level within the framework of a multisector model. See Michael C. Lovell, "Buffer Stocks, Sales Expectations, and Stability: A Multi-sector Theory of the Inventory Cycle," Cowles Foundation Discussion Paper No. 89, March 16, 1960; *Econometrica*, forthcoming.

begin to increase. In the first quarter in which an obligation is let, durable manufacturing inventories are augmented by roughly 10 cents for every dollar of new obligations. When payments are actually made there is a corresponding tendency for inventories to decrease in magnitude. The analysis of the effects of procurement upon durable inventories appears in part III, section 2.

4. *An investigation of the proposition that high interest rates have a tendency to curtail manufacturing inventory investment proved inconclusive.*—Changes in sales and unfilled orders but not interest costs appear as important determinants of inventory investment. This does not mean that monetary policy might not be utilized to influence other components of effective demand so as to help offset swings in inventory investment. It was disappointing not to find evidence suggesting that manufacturing inventory investment might be directly influenced through monetary controls. This issue is discussed in part III, section 3.

5. *Price speculation is not important.*—There is no evidence in support of the hypothesis that the magnitude of manufacturing inventories is influenced by speculation in stocks of purchased materials in advance of anticipated price changes. Part IV, sections 2 and 4, presents the evidence.

6. *Manufacturers do not appear to make large systematic errors in anticipating future sales volume.*—When the behavior of data on inventory investment is considered in conjunction with series on sales and unfilled orders it appears that if manufacturers do have a systematic tendency to underestimate changes in sales volume, it is by only a very small percent. While it is conceivable that revisions of production plans within the quarterly observation period utilized in the study serve in part to hide errors made by manufacturing firms in anticipating sales volume, this means that the effective bias in predicting sales volume is small. This conclusion is based on material in part III and in part IV, sections 3 and 4.

All of these conclusions must be regarded as tentative in nature, as will be apparent from a careful reading of the detailed analysis presented in later sections of this study paper. Investigation of inventory movements is a very difficult task, partially because of difficulties connected with obtaining appropriate data. A principal problem involves the suitable deflation of book value inventory figures. Because of the diverse and complex nature of the accounting practices followed by firms in evaluating their inventory, a very complicated procedure must be followed in adjusting book value inventory figures to a meaningful constant dollar basis. A casual inspection of published book value figures on the current inventory position is likely to be misleading, particularly during periods of rapid price movements. Price increases may result in an increase in book value figures at the very time that an actual reduction in the physical magnitude of stocks is taking place.[3]

[3] An examination of the ratio of book value inventories to dollar sales may also be misleading. In the first place, the ratio is distorted by price changes; book value inventory data respond in an essentially different way from sales figures to changes in the value of the dollar because of differences in composition, turnover lag, LIFO accounting, etc. In the second place, the actual inventory/sales ratio, undistorted by price changes, can hardly be regarded as a parameter of the equation determining actual inventory or more than a rough measure of the discrepancy between equilibrium and actual inventories. For factors determining the inventory/sales ratio see sec. III, equation 2. It would be most helpful in evaluating current inventory movements if the National Income Division of the Department of Commerce could make available on an industry by industry basis estimates of inventory investment in constant dollars or at least data by industry of changes in book value inventory less the inventory valuation adjustment. While such estimates could hardly be expected to be precise, they would probably be less misleading than book value figures unadjusted for the distortion of price changes.

A second problem hampering econometric investigation of inventory behavior is the lack of suitable cross section data on a confidential basis for research purposes reporting at monthly or quarterly intervals the movement of inventories, sales and other related variables at the level of the individual firm. Successful investigation of consumption and durable investment behavior required data on individual households and firms.[4] A complete understanding of inventory behavior will be obtained only when cross section data are made available on a confidential basis for research purposes.

PART II. DELAYED ADJUSTMENT BUFFER-STOCK INVENTORY BEHAVIOR

In this section the details of the delayed adjustment buffer-stock inventory model are presented. This model constitutes the theoretical framework for the econometric investigation of manufacturers' inventories. A series of graphs for a hypothetical firm is presented in order to facilitate an understanding of the model.

For ease of reference the symbols utilized in this paper are listed below:

- X_t sales during period t.
- $\hat{X}_t$ anticipated sales during period t (held at the end of period $t-1$ when the level of production in period t is determined).
- ΔX_t change in sales, X_t-X_{t-1}.
- H_t inventories at the end of period t.
- ΔH_t inventory investment, H_t-H_{t-1}.
- I_t finished goods inventory.
- S_t stocks of purchased materials and goods in process.
- H^p_t inventory planned for the end of period t.
- H^e_t the equilibrium level of inventory for sales volume X_t.
- $\hat{H}^e_t$ equilibrium inventory if anticipated sales materialize, so that $\hat{X}_t=X_t$.
- U_t unfilled orders.
- ΔU_t the change in unfilled orders, U_t-U_{t-1}.
- O^D_t Department of Defense obligations (new orders) for durable goods.
- E_t Department of Defense expenditures for durable goods.

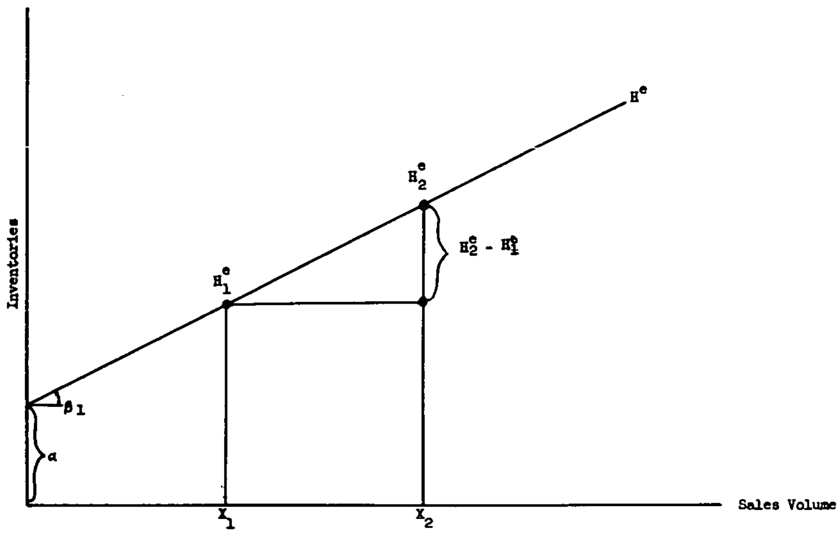
The most elementary form of the acceleration principle relates the desired level of stocks to sales volume. On graph 2 sales in the current quarter, X_t, are plotted on the horizontal axis and end of period inventories, H_t, vertically. The upward slope of the equilibrium inventory line, H^e, implies that more inventories are desired at higher sales levels.[5]

---

[4] Most notable of the cross section studies of investment behavior are John R. Meyer and Edwin Kuh, "The Investment Decision," Harvard University Press, 1957, and Robert Eisner, "A Distributed Lag Investment Function," *Econometrica*, January 1960.

[5] The desired inventory line is not drawn through the origin. Its positive intercept might result from "pipeline" inventories. On an assembly line or in a natural gas pipeline the level of stocks is virtually independent of the speed of production. In a retail store, a basic inventory for purposes of display has to be maintained regardless of the particular level of current sales. A nonzero intercept might also be encountered empirically if the true equilibrium relationship is only approximated by a straight line; it might be the consequence of aggregation over firms and over industries.

Graph 2. The Equilibrium Inventory Relationship



The equation for this relationship is

$$(1) \qquad H_t^e = \alpha + \beta_1 X_t, \qquad \beta_1 \geq 0.$$

If sales during the first quarter are X_1, the desired level of stocks is H_1^e; if sales volume then increases to X_2 during the next quarter, equilibrium stocks are H_2^e. The gap $H_2^e - H_1^e$ represents the level of inventory investment that would be required if inventories were always to be maintained at the equilibrium level. A moment's thought reveals that the conjecture that stocks are always in this simple equilibrium relationship to sales implies that inventory investment is proportional to changes in sales volume.

Empirical evidence long ago led to the rejection of the simple version of the acceleration principle that maintains that inventory investment is proportional to changes in sales.[6] Errors made by firms in anticipating future sales and delayed adjustment behavior suffice to explain departures of inventories from their equilibrium level. After discussing these two factors a more complicated form of the equilibrium inventory relationship will be examined.

Consider first the behavior of finished goods inventory. The discussion is illustrated on graph 3. In period 1 inventories are at level H_1, not necessarily the equilibrium level. The gap $H_1^e - H_1$ represents the inventory discrepancy, the gap between equilibrium and actual inventories. Now suppose that at the end of the first quarter when the level of output of finished goods for the second quarter is being planned, sales of $\hat{X}_2$ are anticipated. If output of $\hat{X}_2$ were to be produced, and if sales anticipations were precisely fulfilled, the gap between equilibrium and actual inventories at the end

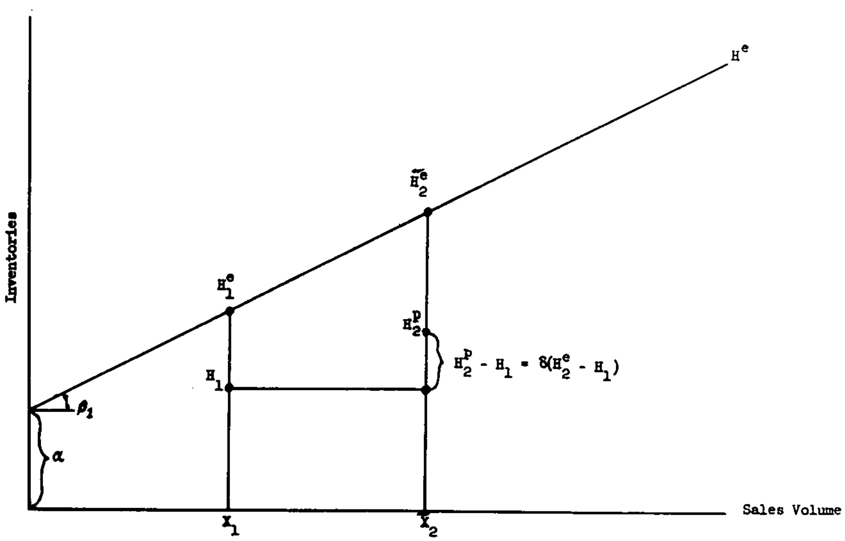
[6] Cf. Moses Abramovitz: "Inventories and Business: Cycles, With Special Reference to Manufacturers Inventories," New York, National Bureau of Economic Research, 1950.

of the second quarter would be $\tilde{H}_2^e - H_1$. In practice, of course, the firm may be expected to produce more than anticipated sales in order to fill this gap, at least partially. If output were $\hat{X}_2 + \tilde{H}_2^e - H_1$, and if there were no errors in anticipating future sales, the discrepancy between actual and equilibrium inventories would be completely eliminated. Costs of changing the level of production and storage capacity and uncertainty as to future sales volume partially explain why firms do not in actual practice attempt to eliminate shortages of finished-goods inventory completely within a single quarter. When finished-goods inventory of seasonal merchandise is excessive, on the other hand, difficulty may be encountered in reducing stocks to the equilibrium level. On graph 3 the gap between the symbol H_2^p representing planned inventory and H_1 indicates the extent to which firms intend to adjust their inventories toward the equilibrium level. If it is postulated that firms, on the average, plan to eliminate a certain fraction δ of the gap between current and anticipated desired inventory, then the level of inventory planned on the basis of sales anticipations $\hat{X}_2$ is specified by the equation

$$(2) \qquad H_t^p = H_{t-1} + \delta(\alpha + \beta_1 \hat{X}_t - H_{t-1}) = (1-\delta)H_{t-1} + \delta(\alpha + \beta_1 \hat{X}_t),$$

where δ may be termed the "reaction coefficient." Production required to fulfill these plans must then be set at $H_t^p - H_{t-1} + \hat{X}_t$.

Graph 3. Delayed Adjustment Inventory Behavior



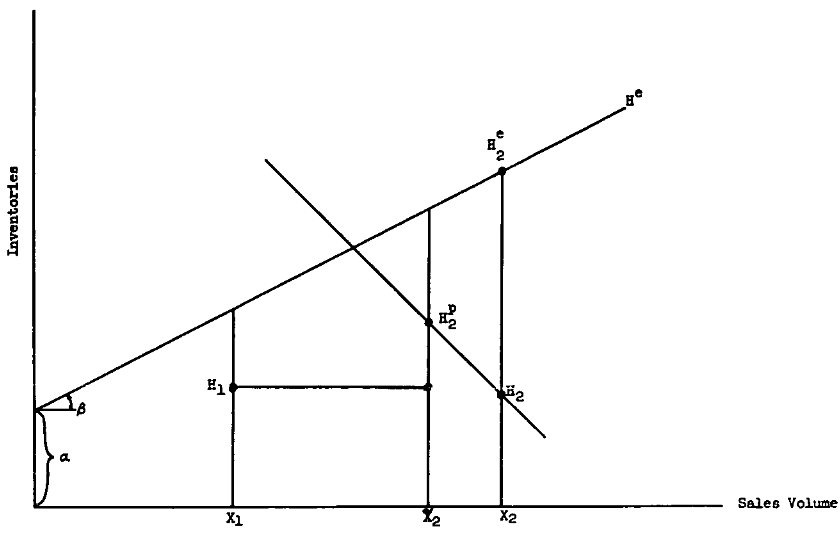
The preceding analysis rested upon the supposition that no errors were made in anticipating future sales volume. Graph 4 shows how errors in predicting sales complicate the picture. It is assumed that the firm adjusts quantity rather than price when an error is made in predicting demand. If the original sales forecast was pessimistic so that actual sales in period 2 turn out to be above $\hat{X}_2$, the unanticipated sales must be met by drawing down inventory below the planned level. Conversely, if actual sales fall short of anticipations, inventories will be built up above the planned level by the excess of anticipated over actual sales.[7] The negatively sloped line indicates the dependence of actual inventory at the end of the second quarter upon how market conditions actually develop, given sales anticipations of $\hat{X}_2$. If actual sales are X_2, inventories at the end of the second quarter are at point H_2 on this line. The equation for actual inventory is

$$H_t = H^p_t + \hat{X}_t - X_t = \delta\alpha + (1+\delta\beta_1)\hat{X}_t - X_t + (1-\delta)H_{t-1}. \tag{3}$$

Subtracting H_{t-1} from both sides of this equation yields the expression for inventory investment

$$\Delta H_t = H_t - H_{t-1} = \delta\alpha + (1+\delta\beta_1)\hat{X}_t - X_t - \delta H_{t-1}. \tag{4}$$

Graph 4. Errors in Anticipating Sales



The flexible accelerator model has been explained in terms of finished goods inventory. Stocks of purchased materials and goods in process may also be analyzed within essentially the same framework. After all, lags in the delivery of purchased materials when production levels increase, together with possible economies arising from quantity ordering, would explain why an immediate adjustment of this category of inventory to its equilibrium level is not attained within any single quarter. Errors in anticipating sales volume, on the other hand, are of secondary importance in explaining purchased material and goods in process inventory. The material reprinted as part IV of this study paper establishes that not too much information is sacrificed when data restrictions prevent the separate analysis of inventories by stage of fabrication.

[7] In Part IV, section 3, the complications arising from the fact that there may be some flexibility in production are considered.

Before turning to the empirical estimates it is necessary to consider one additional complication. The equilibrium level of stocks may depend upon other factors besides output. In particular, the detailed analysis of individual industry data covering the postwar period through 1955, reprinted as part IV, suggests that unfilled orders should be included in equation (1), the function explaining equilibrium inventory. Although entrepreneurs may have reasonably precise plans for production during the next quarter, they must recognize that delivery lags and the costs of adjusting stocks to the equilibrium level prevent the attainment of the precise level of inventory most appropriate for the scale of current output. If firms consider a somewhat longer planning horizon in determining their inventory position, they may regard unfilled orders as constituting an established demand. Unfilled orders may represent an actual committal to deliver at some future date.

The flexible accelerator buffer-stock inventory model may be subjected to both theoretical and empirical test. One may ask whether the theory is compatible with the assumption of profit maximization; Edwin Mills has demonstrated appropriate conditions under which this type of firm behavior coincides with that derived from the assumption that the firm maximizes profit.[8] A second source of a priori information restricting the theoretical framework to be utilized in empirical investigation is obtained by asking whether the dynamic implications of this type of firm behavior are reasonable. An investigation reported elsewhere establishes within the framework of a multisector dynamic model that an accelerator type of inventory behavior is compatible with stability for reasonable values of the parameters only under restrictive conditions. Stability requires both errors of expectations and a reaction coefficient less than unity.[9] The importance of these two complications to the basic accelerator model has thus been established on theoretical grounds. The validity of the theory may also be tested empirically. If one obtained an estimate of the reaction coefficient that was either negative or greater than unity the usefulness of the theory would be subject to serious question. It would hardly be reasonable to obtain a negative estimate of the marginal desired inventory coefficient. In the empirical investigations involving the application of the theory to manufacturers' inventories, both these restrictions upon the magnitudes of the coefficients were satisfied.

PART III. THE EMPIRICAL INVESTIGATION, 1948–60

The analytical framework discussed in part II is appropriate for the econometric investigation of postwar inventory behavior. Quarterly data deflated to a 1954 price base extending from the second quarter of 1948 through 1960 were utilized in the investigation reported in this section. Details of the various time series utilized are presented in an appendix.

It is necessary to preface the estimates with a remark about the difficulty of measuring actual expectations. No quarterly data on

[8] E. S. Mills, "Expectations, Uncertainty, and Inventory Fluctuations," Review of Economic Studies, XXII (1954–55), pp. 15–23.

[9] Michael C. Lovell, Buffer Stocks, Sales Expectations, and Stability: A Multisector Theory of the Inventory Cycle, Cowles Foundation Discussion Paper No. 89, Mar. 16, 1960. Econometrica, forthcoming.

actual expectations covering the 1948 through 1960 period are available. Computing the regressions for a shorter time period in order to utilize available expectations data would involve the sacrifice of much information about the behavior of the other series and might well result in less precise estimates. It seems more appropriate to conjecture that manufacturing firms on the average predict a specific but unknown fraction ρ of actual changes in sales. This conjecture is discussed in detail in the material in part IV, section 3, of this study paper. There it is shown that the assumption permits the following equation to be derived from equation (4) of part II when unfilled orders U_t are included as a determinant of equilibrium inventory.

$$(1)\quad H_t=\delta\alpha+\delta\beta_1X_t-(\delta\beta_1+1)\rho\Delta X_t+(1-\delta)H_{t-1}+\delta\beta_2U_t+\delta\beta_3\Delta U_t.$$

The problem of measuring sales expectations is circumvented by inserting the actual change in sales into the regression equation. An equation for the inventory/sales ratio may be obtained from this last expression by dividing both sizes by X_t

$$(2)\quad \frac{H_t}{X_t}=\frac{\delta\alpha}{X_t}+\delta\beta_1-(\delta\beta_1+1)\rho\left(\frac{\Delta X_t}{X_t}\right)+(1-\delta)\left(\frac{H_{t-1}}{X_t}\right)+\delta\beta_2\left(\frac{U_t}{X_a}\right)+\delta\beta_3\left(\frac{\Delta U_t}{X_t}\right).$$

It is clear that the inventory/sales ratio depends upon the magnitude of any error made in anticipating sales, actual sales volume, and the ratio of last period's inventory to current sales; it can hardly be regarded as a parameter of the system. For purposes of parameter estimation the equation for inventory investment obtained by subtracting H_{t-1} from both sides of (1) was utilized

$$(3)\quad \Delta H_t=\delta\alpha+\delta\beta_1X_t-(\delta\beta_1+1)\rho\Delta X_t-\delta H_{t-1}+\delta\beta_2U_t+\delta\beta_3\Delta U_t.$$

1. The basic equation

Estimates of the parameters of the model obtained by the method of least squares are presented below for durable, nondurable, and total manufacturing.

Total manufacturing:

$$\Delta H_t=\underset{(0.62)}{-1.47}\underset{(.035)}{-.165^{**}H_{t-1}}\underset{(.024)}{+.122^{**}X_t}$$
$$\underset{(.0376)}{-.0118\Delta X_t}\underset{(.0054)}{+.0138^{**}U_t}\underset{(.0240)}{-.0021\Delta U_t}$$
$$R^2=0.53$$

Durable manufacturing:

$$\Delta H_t=\underset{(0.55)}{-1.55^{*}}\underset{(.033)}{-.115^{**}H_{t-1}}\underset{(.033)}{+.127^{**}X_t}$$
$$\underset{(.0441)}{-.0015\Delta X_t}\underset{(.0053)}{+.0087U_t}\underset{(.0209)}{+.0465^{*}\Delta U_t}$$
$$R^2=.57$$

Nondurable manufacturing:

$$\Delta H_t = -\underset{(.409)}{.508} - \underset{(.039)}{.0823^*}H_{t-1} + \underset{(.0161)}{.0426^*}X_t$$
$$- \underset{(.0331)}{.0285}\Delta X_t + \underset{(.046)}{.254^{**}}U_t - \underset{(.058)}{.356^{**}}\Delta U$$
$$R^2 = .66$$

It is to be observed that all estimates of the reaction coefficients, the δ's, are greater than zero but less than unity, as is required by the theory. The estimated value of δ of 0.165 for total manufacturing implies that manufacturers adjust their inventories only a little more than 16 percent of the way toward the equilibrium level within a single quarter. The total manufacturing estimate of 0.122 for $\delta\beta_1$ implies that a dollar increase in sales, other things being equal, leads to 12 cents worth of inventory being accumulated in the current quarter. This is only the immediate impact. The estimated slope of the equilibrium inventory line is

$$0.75 = \frac{0.122}{0.165}.$$

If inventories were initially in equilibrium, a dollar increase in sales, other things being equal, would eventually lead to the accumulation of roughly 75 cents worth of additional inventory. A very low estimate of the coefficient for the change in sales variable appears in all three regressions; as explained in part IV, this implies that there is little or no systematic tendency for manufacturing firms to underestimate changes in sales volume. The total manufacturing estimate of 0.0138 for the coefficient of U_t implies that an increase in the backlog of unfilled orders has only a slight immediate impact upon manufacturers' inventory holdings; when the coefficient of unfilled orders in the regression is divided by the estimate of δ, the resulting coefficient of

$$.085 = \frac{0.0138}{0.165}.$$

may be interpreted in terms of the graphs of part II as the magnitude of the upward shift in the equilibrium inventory curve resulting from an increase in unfilled orders.

A spurious degree of accuracy must not be attributed to these estimates. The standard errors reported below each parameter estimate may be utilized to obtain a rough index of the precision of the coefficients. A single asterisk indicates that a coefficient is more than twice its standard error in magnitude while a double asterisk is placed by those estimates that are more than three times their standard error, suggesting a higher degree of precision. Only a rough indication is provided by this procedure as the error terms are probably autocorrelated.[10]

A second index as to the lack of precision of the estimates is obtained by comparing them with the corresponding estimates presented in table III of the material reprinted as part IV of this study paper.

[10] The Durbin-Watson statistics for the regressions are 1.30 for total manufacturing, 1.23 for durables, and 1.93 for nondurables, suggesting a high likelihood of autocorrelated error terms for the first two regressions. For a discussion of this problem see pt. IV, sec. 2, footnote 9.

Observations were not available beyond 1955 when these earlier estimates were calculated. An additional 5 years of data, and possibly slight difference in deflation procedure as well, explain the discrepancies that may be observed between the two sets of estimates. While not all the discrepancies are small, it is reassuring that for both sets of regressions all the parameter estimates fall within the restricted range suggested by the buffer-stock theory.

A further complication arises in connection with the possibility that various of the parameter estimates are obtained by a biased procedure. The estimating procedure has a systematic tendency to underestimate the reaction coefficient, the coefficient of lagged inventory. For a further review of estimation problems see part IV, section 2.

A measure of the degree to which the model explains fluctuations in inventory investment is provided by the coefficient of determination. The figure for R^2 of 0.53 for total manufacturing implies that the equation explains only a little more than half the variance in manufacturing inventory investment. A much higher coefficient of determination of 0.90 is obtained if the parameter estimates are utilized to predict the level rather than the change in stocks, a much easier task.[11] It is clear that a little experimentation in adding additional variables to the regression would have served to improve the goodness of fit and provided a higher coefficient of determination. It is equally true that the parameter estimates obtained as a result of such experimentation would not necessarily be more precise; the equation itself might not prove more accurate for prediction purposes.[12]

2. The impact of defense procurement

The impact of Department of Defense procurement upon the general level of prices and overall economic activity is an issue of growing importance. An investigation of the effects of defense procurement activities upon manufacturers' inventories is an important facet of this problem. Department of Defense "obligations" is an accounting concept referring to the value of contract placements and other work undertaken during the current quarter. From the point of view of the manufacturer receiving the contract, obligations constitute a new order. Defense expenditures reflect actual payments made by the Defense Department against obligations previously incurred. For the manufacturer receiving payment, expenditures are an item in total sales. At best only a first approximation of the impact upon manufacturers' inventories of defense obligations and expenditures is provided by the coefficients of new orders and sales respectively. It seems reasonable to suppose that defense procurement is rather different in its impact upon the supplier from ordinary civilian orders; they differ in composition and may require extensive research and develop-

[11] A more complete explanation of the effect of the choice of the dependent variable is provided in pt. IV, sec. 2, fn. 7.

[12] A check as to the validity of the regression equation would be provided by utilizing it to predict outside the period for which data were utilized in estimating the parameters of the equation. Because data on inventories are subject to large-scale revision, it is hardly advisable to utilize preliminary data now available for 1961 for this task as errors of observation might be erroneously attributed to the model. Instead, the equation was reestimated over a part of the sample period and the parameters obtained over the shorter period utilized in predicting inventory investment for the quarters excluded from the regression. When observations from the second quarter of 1948 through the second quarter of 1950 were excluded from the regression and the level of inventory investment "predicted" for this period, there was a systematic tendency to understate the degree of inventory disinvestment that actually occurred in the 1949 recession. Although all nine prediction errors were negative in sign, they were not larger in magnitude than one would expect on the basis of the coefficient of determination; while five of the nine errors were more than one standard error of the estimate in magnitude only one was more than two times the standard error of the estimate. The coefficient of determination thus proves to be a fairly accurate indication of the ability of the buffer-stock flexible accelerator inventory model to predict outside the sample period.

ment expenditure. Data starting with the Korean war and extending to the present for both obligations and expenditures permit a more precise estimation of the impact of defense procurement upon inventory investment.[13] It was not possible to obtain a separate reaction coefficient for defense items.

Here are the estimates for the various parameters of the buffer-stock inventory model complicated by the inclusion of data on Department of Defense obligations O_b and expenditures for durable goods E.

$$\Delta H_t = -\underset{(1.36)}{4.01^*} - \underset{(.0450)}{.0683} H_{t-1} + \underset{(.046)}{.184^{**}} X_t + \underset{(.0534)}{.0298} \Delta X_t - \underset{(.0413)}{.0158} \Delta U_t + \underset{(.0092)}{.0112} U_t - \underset{(.166)}{.295} E_t + \underset{(.060)}{.124^*} O_{bt}$$

$$R^2 = .636$$

The coefficients of this equation must be interpreted with care, as certain items actually appear twice in the equation. Thus, defense obligations are included in the ΔU term as well as separately. The figure 0.124 actually represents an estimate of the differential effect of defense as opposed to other types of changes in unfilled orders.[14] The Department of Defense items may be segregated from the non-Defense items; in the following equation the bars over the symbols for sales and the change in orders indicate that the role of defense procurement has been netted out so that these symbols should be interpreted as referring only to the civilian contribution toward total spending and orders respectively.

$$\Delta H_t = -4.01 - .683 H_{-1} + .184 \bar{X}_t + .0298 \Delta \bar{X}_t - .0158 \Delta \bar{U}_t + .0112 U_t - .111 E_t + .109 O_{bt}.$$

The coefficient for the obligations term suggests that the immediate impact of an increase in obligations of $1 is to augment inventories by one-tenth of that amount. Defense procurement has an immediate impact upon inventories during the quarter in which the obligations are let, even though considerable time may elapse before either delivery or actual payment for the goods takes place. Conversely, during the quarter in which payment is made for the goods durable manufacturing inventory is reduced proportionately, offsetting the initial effect.

These figures yield only the initial impact of defense obligations and expenditure. In order to derive the ultimate effect upon inventories of procurement activity it does not suffice to divide through by the estimate of the reaction coefficient. The effect of defense procurement upon nondefense sales and orders must also be considered. When new workers are hired in order to produce the items needed for defense purposes additional orders and sales are generated as they spend funds they earn in defense production. This leads to the accumulation of additional inventory. A much more complicated

[13] Unfortunately, data on the backlog of Department of Defense unpaid obligations, the counterpart of the unfilled orders concept, are available only for the period 1954 through 1960. This series was excluded from the regression reported here in order that the important Korean war period could be included within the study. This is equivalent to assuming that the manufacturer does not differentiate between the defense and civilian backlog of unfilled orders.

[14] The fact that the coefficient of obligations is significant by classical test procedures at the 5-percent level should be interpreted as implying that the differential effect should not be regarded as simply the consequence of sampling error.

framework than that utilized in this investigation would be required to trace through all these effects.

3. The interest rate and inventories

Although economists have frequently argued that inventory investment must be sensitive to changes in the rate of interest, the empirical verification of the proposition has proved to be fraught with difficulties. A fairly close correspondence between total inventory holdings in manufacturing and business loans over the period 1948 through 1955 was observed by Doris M. Eisemann.[15] Albert G. Hart's[16] observations confirm the existence of a similar correspondence in more recent years. It clearly cannot be argued, however, that such association in itself establishes that the monetary authorities, by adjusting the rate of interest, can directly influence the level of inventories. The correspondence might be simply the coincidental movement of essentially independent series during periods of inflation or of general business expansion and contraction. Even if the expansion of bank loans was the consequence of inventory accumulation, it is still possible that the behavior of inventories is determined by real factors and is insensitive to either the cost or the availability of credit.

Two related arguments may explain why manufacturing firms might not be induced to enlarge their inventory above the level suggested by nonmonetary considerations when funds become generally available on easy terms. As Moses Abramovitz's calculations suggest, the typical manufacturing firm may find that the interest costs involved in carrying inventory are small relative to warehousing expense and loss due to shrinkage.[17] In addition, day-to-day inventory decisions for many firms may quite typically be made at a routine level on the basis of fairly simple mechanical rules specifying such factors as reorder points and optimal order quantity; while the cost of funds may have entered into the equation utilized in formulating the decision rule, the instructions can hardly be revised with every change in the interest rate.

The buffer-stock inventory model provides a framework for the investigation of the hypothesis that interest rates affect inventory investment directly. If interest rates are to influence inventory investment, they must operate by affecting the equilibrium level of inventories; the higher the rate of interest the lower must be the equilibrium level of stocks. Given the volume of sales, the magnitude of the stocks inherited from the preceding quarter, unfilled orders, and so forth, the firm is conjectured to carry less inventories over into the next quarter the higher the interest rate. A negative relationship observed when the rate of interest is introduced into the buffer-stock regression equation might be interpreted as giving definite support to the hypothesis that monetary policy can be successfully employed to mitigate the inventory cycle.

The results were quite disappointing. When the business loan rate was added to the other variables in the equation, it invariably had a significantly positive coefficient. This was true for both durable and

[15] Doris M. Eisemann, "Bank Credit and Inventory Cycles," American Statistical Association, Proceedings of the Business and Economics Statistics Section, Washington, 1957, pp. 75–86.

[16] Albert G. Hart, "Making Monetary Policy More Effective," United States Monetary Policy, The American Assembly, Columbia University, 1958, pp. 186, 187.

[17] Moses Abramovitz, op. cit., pp. 125, 126.

nondurable manufacturing. The positive association between inventories and the interest rate may be explained as follows. Changes in sales and possibly orders as well lead to an expansion in the level of inventories that firms want to carry at the current rate of interest. This causes the demand schedule for loanable funds to finance these holdings to shift to the right along a comparatively stable supply schedule for loanable funds. Although a positive association between interest rates and inventories results, this has little bearing on the issue of how sensitive inventory investment is to interest rate changes.

Simultaneous equation estimation techniques might be applied in order to untangle the complex interrelationships, but the task of estimating a supply schedule for loanable funds is clearly beyond the scope of this investigation. An alternative approach would be possible if data were available for individual firms, provided one could assume that, within the relevant range, the terms on which funds can currently be borrowed by the firm are insensitive to the amount borrowed and depend more on general business conditions. Unfortunately, quarterly or monthly cross section data appropriate for the examination of inventory behavior are not currently available.

A second set of calculations was prepared utilizing the data on individual industries discussed in the material reprinted as part IV of this study paper. Only to the extent that individual industries do not expand their inventories at essentially the same time does an industry-by-industry approach circumvent the simultaneous equation problem mentioned in the preceding paragraph. Rather than recalculate the entire regression with the added interest rate variable, the behavior of the residuals was examined. While this provides a somewhat weaker test, it saved considerable time and computational effort and permitted the consideration of a spectrum of alternative interest rates. The rate on prime commercial paper, the bank rate on short-term business loans, the 3-month bill rate, and the rate on prime bankers acceptances were correlated with the residuals from the various industry regressions. None of the correlation coefficients was significant at the 5-percent level. This second test offered no support to the conjecture that interest rate costs have a direct effect on inventory investment.

Appendix to Part III

A brief description of the various statistical series utilized in the empirical investigation follows:

1. Inventories and inventory investment, both durable and nondurable: These data in price deflated seasonably adjusted form were provided on a quarterly basis by the Office of Statistical Standards through the courtesy of the Joint Economic Committee.

2. Unfilled orders data: Durable and nondurable unfilled orders data published in the Survey of Current Business were deflated with durable and nondurable manufacturing wholesale price indexes published by the Bureau of Labor Statistics in the Monthly Labor Review. Although these data were not deseasonalized, the seasonal pattern is so unpronounced at this level of aggregation as to not materially affect the estimates.

3. Change in unfilled orders: These series were derived by first differencing the deflated unfilled orders data.

4. Durable and nondurable sales data: Deseasonalized data published in the Survey of Current Business and Business Statistics, were deflated with the same price indexes utilized for the unfilled orders data.

5. Department of Defense obligations and expenditures data: Nondurable expenditure and obligation items such as subsistence and petroleum products and clothing are excluded from these series. The data were deflated by the price index utilized for the deflation of durable sales and unfilled orders. These data were published in connection with testimony of Charles J. Hitch, Assistant Secretary, Department of Defense, before the Joint Economic Committee. "January 1961 Economic Report of the President and the Economic Situation and Outlook," Congress of the United States, 87th Congress, 1st session. U.S. Government Printing Office, 1961, pages 667 and 668.

These data were not fully comparable with the series utilized in the earlier study of manufacturing inventory reprinted as part IV of this study paper. For that study the orders and sales figures were deflated on essentially a two-digit industry basis and aggregated to form nondurable and durable series. This procedure has an advantage in that it does not rest on the implicit assumption that changes in the interindustry composition of sales and orders were negligible during the period under investigation.

76626 O—61—pt. II——10

PART IV

Econometrica, Vol. 29, No. 3 (July 1961)

MANUFACTURERS' INVENTORIES, SALES EXPECTATIONS, AND THE ACCELERATION PRINCIPLE[1]

BY MICHAEL LOVELL

The response of manufacturers' inventory holdings to changes in the volume of sales and the backlog of unfilled orders is examined on a quarterly basis for the period 1948–55 within a buffer-stock flexible accelerator framework. The hypothesis that manufacturers successfully hedge against increases in the price of purchased materials, enlarging their stocks in advance of actual price increases, is rejected. By introducing explicitly the impact of prediction errors it is possible to infer that manufacturers tend to underestimate actual changes in sales volume, but by a surprisingly small amount. An analysis of discrepancies between desired and actual inventory holdings reveals that manufacturers tolerated sizable deficiencies in stocks throughout the Korean conflict.

1. INTRODUCTION

IN HIS METICULOUS investigation of the behavior of manufacturers' inventories during the interwar period, Moses Abramovitz considered the most elementary of accelerator models, the hypothesis that maintains [**1**, p. 20]:

> Manufacturers and merchants are both desirous and able to maintain inventories in constant ratio to their output or sales, . . . (that) inventories vary directly and proportionately with output.

Abramovitz found that this most simple concept of the accelerator was not consistent with observed behavior. Contrary to hypothesis, actual inventory investment does not lead peaks and troughs of output; inventory investment is not proportionate to the rate of change in output.

The greater variety and detail of data for the postwar period permit us to consider more complex versions of the acceleration principle. One modification involves the flexible accelerator concept originally presented by Richard Goodwin [**6**], a model that assumes that business firms attempt only a partial adjustment of stocks to their equilibrium level during each

[1] This paper constitutes a revision of certain materials appearing in my doctoral dissertation [**12**, Ch. 3], a research project supervised by Wassily Leontief. I am indebted to the Earhart Foundation, the Social Science Research Council, and the Cowles Foundation for Research in Economics for financial support. Computations were performed at the Littauer Statistical Laboratory, Harvard University, and the Yale Computing Center. Albert Beaton, William Locke Anderson, Gerald Kraft, and Harold Watts, provided helpful advice in connection with data processing problems. I wish to express appreciation for the advice and criticism of Leo Bakony, George M. Cobren, James Henderson, Ruth Mack, Edwin S. Mills, Arthur M. Okun, Richard Porter, and Thomas M. Stanback, Jr.; but I must bear full responsibility for any remaining errors.

production period.[2] A second complication of the basic accelerator involves the hypothesis of Lundberg [13] and Metzler [16] that errors made by firms in forecasting future sales generate discrepancies between the actual and the desired level of inventories. In this paper both these complications are considered in investigating the appropriateness of the accelerator as a mechanism explaining inventory behavior. We also consider the possibility that unfilled orders and expected price changes as well as the volume of activity affect the desired level of stocks.

Both the flexible accelerator model of Goodwin and the buffer stock type of inventory model advanced by Lundberg and Metzler explain the generation of discrepancies between actual and "equilibrium" or "desired" inventories. Estimates of the parameters of an accelerator model incorporating the complications introduced by these economists are utilized to compute a series of surplus inventories, the deviation of actual stocks from their equilibrium levels. These estimates of surpluses and deficiencies in manufacturers' inventory holdings are presented in the concluding section of this paper.

The task of investigating dynamic inventory phenomena is complicated by the difficulties involved in obtaining appropriate data based on observations collected at more frequent than yearly intervals. Since the planning horizon of the firm is surely shorter than a year for decisions involving output adjustment and inventory, annual data will not do. I have not succeeded in obtaining appropriate cross-section data on individual firms. This study is based on quarterly time series data at a fairly high level of aggregation. For total durable and total nondurable sectors of manufacturing, the Office of Business Economics, Department of Commerce, publishes data broken down by stage of fabrication into purchased materials, goods in process, and finished goods categories. But when we turn to Commerce data for individual industries we find that the more detailed breakdown as to industry must be paid for by sacrificing the stage of fabrication classification. Nevertheless, these are the data we must use. Thomas M. Stanback, Jr. of New York University kindly provided deflated, deseasonalized sets of the Commerce inventory data for the years 1948 to 1955.[3] W. H. Locke Anderson of Harvard University collaborated with me in deflating comparable sets of sales and unfilled orders data from Office of Business Economics series published by the Department of Commerce in the *Survey of Current Business*. Output is

[2] Goodwin's flexible accelerator is related to Chenery's overcapacity principle [2].

[3] Stanback has published a description of the deflation procedure in his excellent study of postwar inventory behavior [19, p. 91]. For a general discussion of the difficulties involved in deflating inventories see Cobren [3]. He describes the deflation procedure utilized by the National Income Division, Department of Commerce, a procedure similar to that employed by Stanback.

defined as sales plus any increase in finished goods inventory, rather than measured independently.

We shall first consider two separate accelerator models of inventory behavior. The first of these is appropriately applied to stocks of purchased materials and goods in process; the second to finished goods inventory. These models are tested against the durable and nondurable sector data in the next two sections of this paper. Then, in Section 4, we combine these two models into a single equation explaining the behavior of total inventory holdings in each of the five component durable goods industries.

2. STOCKS OF PURCHASED MATERIALS AND GOODS IN PROCESS

In applying the principle of acceleration to stocks of purchased materials and goods in process it seems reasonable to relate stocks at the *beginning* of the production period, S_t, to output forthcoming *during* the period, Q_t. If the relation is linear, the equilibrium level of stocks, S_t^e, may be represented by the equation

$$S_t^e = \alpha + \beta Q_t . \tag{2.1}$$

The coefficient β is the "marginal desired inventory coefficient."

Only the simplest version of the accelerator hypothesis asserts that entrepreneurs always succeed in maintaining stocks at the equilibrium level. Goodwin's flexible accelerator concept provides one explanation of departures of stocks from their equilibrium level.[4] Now suppose output is initially at Q_1 with stocks at S_1, not necessarily the equilibrium level. If output then increases to Q_2, the simple accelerator hypothesis implies that stocks would be adjusted immediately to the new equilibrium level $S_2^e = \alpha + \beta Q_2$, that supplies acquired would exceed immediate production requirements by $S_2^e - S_1$. But Goodwin supposes that an immediate adjustment of stocks is not attempted. Entrepreneurs are assumed to make only a partial adjustment of stocks to the equilibrium level each period. This may be due to costs involved in changing the level of stocks. It may also stem from problems concerned with the heterogeneous nature of stocks, the infrequent intervals at which certain items are ordered, and so forth. Certain categories of purchased materials may be held by the firm in relatively large stocks as a result of recent deliveries of carload orders; they will be liquidated only gradually as production demands require their utilization. On the other hand,

[4] We shall regard production plans as reasonably firm at the beginning of periods so that the buffer stock motive can be neglected in the study of purchased materials and goods in process stocks; but this alternative explanation of departures of stocks from their equilibrium level will be considered in the examination of finished goods inventory behavior.

a rapid increase in the holdings of certain items warranted by a higher level of activity would be costly if a premium has to be paid for fast delivery. Such factors as these may explain why only a partial attempt to adjust inventories to the level dictated by equation (2.1) may be attempted in any one period. The assumption that actual inventory investment, ΔS_t, is only a fraction of that required to adjust stocks to the equilibrium level is reflected in the equation

$$\Delta S_t = S_t - S_{t-1} = \delta(S_t^e - S_{t-1}) = \delta\alpha + \delta\beta Q_t - \delta S_{t-1} . \quad (2.2)$$

Adding S_{t-1} to both sides of this equation yields

$$S_t = \delta\alpha + \delta\beta Q_t + (1-\delta) S_{t-1} . \quad (2.3)$$

This generalization reduces to the simple accelerator hypothesis for the special case in which δ, the reaction coefficient, is unity.

The flexible version of the acceleration principle is related to Ruth Mack's notion of "passive inventory investment." She asserts [**14**, p. 480]:

> Passive inventory investment or disinvestment takes place in part because plans about the proper size of stocks are hardly precise figures; instead they are ranges, and variation within the range or band is a matter of indifference.

But our equation implies that while there may be a precise notion as to the appropriate size of stocks, business firms' investment in stocks is "passive" in the sense that they are not overly concerned with obtaining a rapid adjustment to that level when changes in business conditions cause stocks to depart from it.[5]

Within the framework of the flexible accelerator model three additional factors that may complicate the determination of the size of stocks of purchased materials and goods in process deserve consideration.

(a) *Price speculation.* Manufacturers may attempt to hedge against anticipated increases in the price of inputs by adjusting their inventory position, purchasing additional stocks when price rises are expected and reducing the level of stocks when price reductions are anticipated. An adequate test of the prevalence of inventory speculation would require knowledge of *expected* price changes. Unfortunately, data on price anticipations are not available. While a possible approach would be to attempt to incorporate within the model assumptions concerning the actual structure by which anticipations of future price changes are generated, there are a

[5] Franco Modigliani has commented upon the relation between the flexible accelerator and Ruth Mack's concept of passive inventory investment. He quite rightly suggested that the speed of adjustment may depend upon the size of the discrepancy between desired and actual inventories [**18**]. One might well conjecture that it would be difficult to distinguish passive from delayed adjustment inventory behavior empirically.

vast number of alternative equations based on a naive projection of past price changes, distributive lag relations and so forth, admissible as candidates for this purpose. Rather than attempt on the basis of experimentation with a sample of inadequate size to determine the actual structure by which anticipations are generated, an alternative approach may be followed to the issue of whether a speculation motive partially determines the size of stocks of purchased materials and goods in process. Profitable speculation would involve expanding inventories above the level dictated by other considerations in advance of actual price increases and in reducing them below customary levels prior to actual reductions in price. Including the percentage increase of *actual* prices in the *next* period in the regression amounts to testing the hypothesis that entrepreneurs in fact speculate successfully, accumulating stocks that are larger than would be suggested by purely nonmonetary considerations in advance of actual price increases, and conversely. It assumes that actual changes are correctly anticipated, as is required for profitable speculation. Conversely, if entrepreneurs are the opposite of clairvoyant, speculating on the basis of perverse price anticipations that are negatively correlated with actual developments, the regression coefficient for the actual future price change would be negative.

(b) *Changes in output.* Departures of stocks from the level suggested by the simple accelerator principle may also occur when output is sharply changing. When output is increasing, orders may be placed with suppliers in an attempt to build up stocks, but considerable delays may be involved in obtaining delivery. Consequently, there may be a tendency for stocks of purchased materials to fall below the desired level when output is rising, when $\Delta Q_t > 0$, and conversely. This tendency, which is quite apart from flexible accelerator complications, may be accentuated by the need to measure output only at the final stage of the production process; output is defined as sales plus any increase in finished goods inventories.

(c) *Unfilled orders.* The equilibrium level of stocks may depend on other factors besides output. In particular, we argue that unfilled orders as well as output should be included in equation (2.1). Entrepreneurs may have reasonably precise plans for production in the next period. They may also consider a longer planning horizon in deciding upon the change in stocks to be made in the current period. After all, delivery lags and costs of adjustment will prevent in any case the attainment of the level of stocks that is suggested by current output and speculation considerations. If unfilled orders represent an established demand, indeed a possible committal to deliver at some future date, entrepreneurs may well consider it advisable to carry additional stocks when unfilled orders are large as a hedge against possible shortage and price commitments. In addition, a rise in the backlog of unfilled orders may be expected to lead to an acceleration of production that is felt first in terms of an increase of goods in process rather than a rise in the output of completed commodities. These considerations suggest that stocks of purchased materials and goods in process should be positively related to the backlog of unfilled orders.[6] Conversely, if unfilled orders were only a surrogate measure of the tightness of the markets on which firms purchase their inputs, a negative relationship between orders and stocks would be revealed when unfilled orders at the beginning of the period, U_t, are considered in the regression. These considerations suggest that a more complicated equation than (2.1) describes the equilibrium level of manufacturers' stocks of purchased materials and goods in process:

$$S_t^e = \alpha + \beta_1 Q_t + \beta_2 \Delta Q_t + \beta_3 \left(\frac{p_t - p_{t-1}}{p_t}\right) + \beta_4 U_t \,, \tag{2.1*}$$

where p_t is the level of the price index at the end of period t.

If, following Goodwin, actual stocks are assumed to result from only a partial adjustment of last period's stocks towards the current equilibrium level, we have

$$S_t = \delta[\alpha + \beta_1 Q_t + \beta_2 \Delta Q_t + \beta_3 \left(\frac{p_t - p_{t-1}}{p_t}\right) + \beta_4 U_t] + (1-\delta) S_{t-1} + \varepsilon_t . \tag{2.3*}$$

The residual, ε_t, may be regarded as representing variables omitted from the analysis. It may be observed that an expression for investment in stocks is obtained by subtracting S_{t-1} from both sides of the equation, yielding

$$\Delta S_t = \delta\alpha + \delta\beta_1 Q_t + \delta\beta_2 \Delta Q_t + \delta\beta_3 \left(\frac{p_t - p_{t-1}}{p_t}\right) + \delta\beta_4 U_t - \delta S_{t-1} + \varepsilon_t . \tag{2.4}$$

Since the magnitude of the residual is unaffected, it is immaterial which of these two equations we fit by the method of least squares.[7] The regression coefficients obtained by least squares may obviously be unscrambled in order to obtain implied estimates of the model's parameters.

Do entrepreneurs speculate in stocks? Are unfilled orders a major factor influencing the inventory position of firms? Does a flexible accelerator model provide a reasonable description of observed inventory behavior during the post World War II period? These are interesting questions, but it must be

[6] Stanback observed a close correspondence between turning points for unfilled orders and stocks of purchased materials [19, p. 90]. P. Darling suggests that the change in unfilled orders should be considered in explaining the behavior of total inventory holdings [4].

[7] The magnitude of the regression coefficients and their estimated standard errors will be precisely the same; of course, the multiple correlation coefficient will probably be smaller for (2.4), as the variance of inventory investment is usually larger than the variance of the stock itself.

emphasized that the estimates presented in this paper have been obtained by applying crude least squares procedures to embarrassingly short time series. True, Mann and Wald [15] have demonstrated that if the residuals of a model of the form (2.4) are normally and independently distributed, the least squares procedure will yield maximum likelihood estimates of the equation's parameters. But this is of little consolation, at least for hypothesis testing, as it has been demonstrated by Leonid Hurwicz [8] that the inclusion of the predetermined variable S_{t-1} means that the estimates are biased. Quite apart from this, if the error terms ε_t are autocorrelated, the t distribution may not be appropriately applied in tests of hypotheses about the magnitude of the regression coefficients without adjustment[8] and, in addition, the estimating procedure is not efficient. Perhaps the most crucial limitation is that consideration of a single equation neglects the underlying, possibly simultaneous, interdependent nature of the economy. These difficulties must be kept firmly in mind in examining estimates of the parameters of equations obtained by applying the least squares procedure, particularly in interpreting estimates of the standard errors of the regression coefficients. While the figures reported in the following tables are carried out to four places, this represents a spurious accuracy and is not indicative of precision estimates.

Table I presents the estimates.[9] It is to be observed that all the reaction coefficients, δ, are of the right sign and less than unity. This is encouraging, for one would be at a loss to explain a value of δ outside this range. Indeed, a reaction coefficient of acceptable magnitude is a prerequisite that must be satisfied if any meaning is to be attached to the other coefficients obtained in the regression. It must be admitted that the reaction coefficient for non-durable manufacturing is exceedingly small, indeed almost unreasonably so; this implies that entrepreneurs are little concerned with adjusting inventories to the appropriate level for this sector of the economy. For all sectors

[8] For the case in which predetermined variables are not present Wold [21, pp. 209–213] has derived the appropriate correction procedure; the corrected value of t may be either larger or smaller than that obtained by dividing the regression coefficient by the "standard error" obtained by simple, least squares procedures. An alternative approach is suggested by Klein [10, pp. 85–92].

[9] Least squares estimates of the standard error of the regression coefficients appear below each estimate in parentheses. The coefficient of determination, R^2, applies to the case in which total stocks are taken as the dependent variable. d is a statistic utilized in the Durbin-Watson test for serial correlation of residuals. Let $k = 5$ be the number of explanatory variables, $n = 29$ the number of observations, and $d = \Sigma (e_\tau - e_{\tau-1})^2 / \Sigma e_\tau^2$, where e_τ is the observed residual. Then the hypothesis of no serial correlation is to be rejected at the 5 per cent level if $d < 1.05$ or accepted if $d > 1.84$; otherwise the situation is indeterminate. This test is based on a distribution derived only for equations which do not involve predetermined variables; consequently, it is not strictly valid for our problem. Cf. Durbin and Watson [5].

TABLE I

STOCKS OF PURCHASED MATERIALS AND GOODS IN PROCESS

$$S_t = \delta\alpha + \delta\beta_1 Q_t + \delta\beta_2 \Delta Q_t + \delta\beta_3((p_t - p_{t-1})/p_t) + \delta\beta_4 U_t + (1 - \delta)S_{t-1} + \varepsilon_t .$$

Number of Observations	Total Manufacturing	Total Durables	Total Nondurables
	29	29	29
$\delta\alpha$	4004	1412	—356.0
$\delta\beta_1$	.0620	.0528	.0230
	(.0160)	(.0187)	(.0209)
$\delta\beta_2$	—.0997	—.0803	—.0366
	(.0303)	(.0296)	(.0559)
$\delta\beta_3$	—.3204	.0385	.1481
	(.2061)	(.1732)	(.1208)
$\delta\beta_4$	.0609	.0384	.2207
	(.0053)	(.0041)	(.0514)
δ	.4576	.3628	.0974
	(.0455)	(.0338)	(.0665)
R^2	.993	.994	.970
d	2.273	1.822	2.019
β_1	.1355	.1455	.2361
β_4	.1332	.1058	2.266

the values of the reaction coefficients are sufficiently small to be inconsistent with the simple accelerator hypothesis which maintains that entrepreneurs attempt an immediate adjustment of inventories to the desired level; these estimates of the reaction coefficients lend support to the flexible accelerator concept of Richard Goodwin.[10]

Perhaps the evidence is clearest with regard to speculation. Certainly, there is no support for the hypothesis that manufacturers successfully speculate in stocks of purchased materials and goods in process. The sign for total manufacturing is negative, contrary to hypothesis; while the coeffi-

[10] These estimates of the reaction coefficients for inventories may be compared with idle money balance adjustment figures implied by a recent study by M. Bronfenbrenner and T. Mayer. They report in "Liquidity Functions in the American Economy," *Econometrica*, Vol. 28 (1960), p. 817, three annual adjustment coefficients for the money market ranging in magnitude from 0.28 to 0.46. If the quarterly inventory reaction coefficients of Table I are converted by the equation $1-(1-\delta)^4$ we obtain annual reaction coefficients of 0.34, 0.84, and 0.91 for nondurable, durable, and total manufacturing stocks respectively. Relative to the adjustments of the money market, manufacturers adapt their stocks of purchased materials and goods in process rather rapidly. This touches upon the issue of the speed with which the full impact of fiscal as opposed to monetary policy is felt by the economy.

TABLE II

FINISHED GOODS INVENTORY

$$I_t = \delta\alpha + \delta\beta X_t - (\delta\beta + 1)\varrho\Delta X_t + (1 - \delta)I_{t-1} + \varepsilon_t$$

	Total Manufacturing	Total Durables	Total Nondurables
Number of Observations	30	30	30
$\delta\alpha$	—258.2	—325.8	418.7
$\delta\beta$	.0419	.0550	.0058
	(.0203)	(.0143)	(.0292)
$(\delta\beta + 1)\varrho$	.1315	.0970	.1695
	(.0417)	(.0283)	(.0685)
$1 - \delta$	.8479	.8171	.9351
	(.0649)	(.0523)	(.0858)
δ	.1521	.1829	.0649
ϱ	.1262	.0919	.1685
β	.2755	.3007	.0894
d	1.39	1.33	1.57
R^2	.958	.966	.947

cients are of the right sign for the durable and nondurable sectors, they are exceedingly small relative to their standard errors. While there is no reason for questioning Abramovitz's tentative conclusion that price hedging is not an important phenomenon [1, pp. 127-31], two qualifications to our argument must be mentioned. In the first place, successful speculation on the part of individual firms may involve adjustments in the composition of inventories rather than any change in their aggregate volume; firms may expand their holdings of those commodities whose prices are expected to increase most rapidly and make proportionate reductions in their holdings of other commodities whose prices are expected to fall or at least rise less rapidly. A second qualification also arises from a problem of aggregation. It is possible that while firms within each industry successfully anticipate price changes and adjust the size of their inventory holdings accordingly, the effects of speculation may cancel out when we aggregate over several industries in examining data for total durables and nondurables. Evidence for individual industries presented in Table III below suggests that the negative conclusion concerning speculation is not simply the consequence of aggregating to the durable and nondurable level. While there is no basis for rejecting the conjecture that firms change only the composition rather than the magnitude of their inventory holdings in response to price changes, it may well be that such a restricted form of speculation would be of little import as far as explaining cyclical disturbances in the general level of economic activity is concerned.

TABLE III

TOTAL INVENTORIES

$$H_t = \delta\alpha + \delta\beta_1 X_t + (1-\delta) H_{t-1} - (\delta\beta_1 + 1)\varrho \Delta X_t + \delta\beta_2 U_t + \delta\beta_3 (\Delta p_{t+1}/p_t) + \varepsilon_t$$

Number of Observations	Total Manufacturing	Total Durables	Total Nondurables	Stone, Clay, and Glass	Primary Metal	Transportation Equipment	Machinery	Other Durables
	30	30	30	33	30	31	31	29
$\delta\alpha$	3066.	1032.	–661.0	27.38	–172.8	266.0	751.9	32.77
$\delta\beta_1$	.1672	.1256	.0355	.1085	.0631	.0827	.0347	.1221
	(.0331)	(.0368)	(.0383)	(.0197)	(.0309)	(.0318)	(.0389)	(.0298)
$1-\delta$	.5699	.6756	.9263	.7333	.9446	.6840	.7010	.8055
	(.0514)	(.0448)	(.0653)	(.0508)	(.0533)	(.0542)	(.0432)	(.0500)
$(\delta\beta + 1)\varrho$	+.1697	+.1043	+.1706	+.2343	+.0432	+.0306	+.0697	+.1347
	(.0471)	(.0538)	(.0746)	(.0503)	(.0321)	(.0459)	(.0653)	(.0484)
$\delta\beta_2$	.0618	.0374	.3291	*	.0179	.0319	.0593	*
	(.0076)	(.0062)	(.0823)		(.0115)	(.0049)	(.0067)	
$\delta\beta_3$	–1.6852	–.4991	–.6180	.0025	–.0355	.0057	.0288	*
	(.4101)	(.3311)	(.1847)	(.0200)	(.0346)	(.0842)	(.0668)	
δ	.4301	.3244	.0737	.2667	.0554	.3160	.2990	.1945
β_1	.3887	.3872	.4817	.4068	1.1390	.2617	.1161	.6278
ϱ	.1454	.0927	.1648	.2114	.0406	.0283	.0674	.1200
R^2	.993	.991	.981	.978	.939	.990	.991	.960
d	1.98	1.46	2.23	1.29	1.72	1.13	1.49	0.92

* Lack of data precluded the inclusion of unfilled orders in the stone, clay, and glass regression and of both the percentage price change and unfilled orders for other durables.

The estimates obtained for β_4 suggest that unfilled orders are an important determinant of the level of stocks. To the extent that the backlog of unfilled orders represents an established demand, entrepreneurs increase their stocks of purchased materials as output expands at earlier stages of production. The rather small estimates obtained for the marginal desired inventory coefficient, β_1, reflects the role of the backlog of unfilled orders as well as the scale of output in determining the desired level of purchased materials and goods in process. Nevertheless, it is surprising that in every case β_1 plus β_2 add to a negative figure as this implies that stocks are lower, other things being equal, in advance of increases in output.[11] But with the exception of this peculiarity, which may best be attributed to sampling error,[12] all parameter estimates are in conformity with the flexible accelerator inventory model, modified by the inclusion of unfilled orders.

3. INVENTORIES OF FINISHED GOODS: THE BUFFER STOCK MOTIVE

Planned output is one of the variables considered by the firm in determining the appropriate adjustment of purchased materials and goods in process. The decision concerning the appropriate level of output is in turn based upon anticipated sales and the current inventory of finished goods. The buffer stock inventory model recognizes that the production process is time consuming. It is a modification of the basic accelerator designed to take into account errors made by firms in anticipating future sales. In the form in which the model was originally considered by Lundberg and Metzler, desired end of period inventories, I_t^d, were assumed to be linearly related to sales volume

$$I_t^d = \alpha + \beta X_t \,. \tag{3.1}$$

But actual sales are not known by the firm in advance of output when the production decision must be made. A planned level of inventories, I_t^p, may be defined by substituting anticipated sales, $\hat{X}_t$, into (3.1). Actual end of period inventories will differ from this planned level if sales turn out to exceed the expected volume, and conversely. Consequently, actual end of period inventories, I_t, are given by the equation

$$I_t = I_t^p + (\hat{X}_t - X_t) = \alpha + (1 + \beta)\hat{X}_t - X_t \,. \tag{3.2}$$

[11] Because $\beta_1 Q_t + \beta_2(Q_t - Q_{t-1}) = \beta_1 Q_{t-1} + (\beta_1 + \beta_2)[Q_t - Q_{t-1}]$.

[12] An alternative explanation, in the spirit of the analysis of finished goods inventory that follows, is to assume that observed output, Q_t, is the result of a flexible adjustment, based on subsequent sales experience during the quarter, of the production plan that was utilized in determining the initial stock of purchased material and goods in process.

Lundberg and Metzler explained deviations of actual inventories from the equilibrium level in terms of errors in anticipating future sales.[13]

The buffer stock inventory model is readily married with Goodwin's flexible accelerator. If it is again supposed that the firm attempts only a partial adjustment of its inventory to the new equilibrium level, we have as the equation for the planned level of inventory

$$I_t^p = \delta I_t^e + (1-\delta) I_{t-1}, \qquad 0 < \delta \leqslant 1, \tag{3.3}$$

where δ is the reaction coefficient. Output is set at anticipated sales plus the excess of planned inventory over actual beginning period inventory. Since this production decision is assumed irrevocable, any subsequent deviation of actual sales from the anticipated volume leads to a corresponding departure of actual inventory from the planned level.[14] As with the simple buffer stock inventory model, actual inventories will deviate from the planned level by any excess of anticipated over actual sales:

$$\begin{aligned} I_t = I_t^p + \hat{X}_t - X_t = \delta I_t^e + (1-\delta) I_{t-1} + \hat{X}_t - X_t = \\ \delta(\alpha + \beta \hat{X}_t) + (1-\delta) I_{t-1} + \hat{X}_t - X_t . \end{aligned} \tag{3.4}$$

This is the flexible accelerator version of the buffer stock inventory model. For the special case in which $\delta = 1$ it reduces to the more elementary equation considered by Lundberg and Metzler, (3.2) above.

Investigation of the buffer stock motive of inventory behavior necessarily involves complications concerning the nature of sales expectations. Since sales expectations are not an item in the accounting records of firms they may be measured directly only by *ex ante* questionnaires; consequently, it is most difficult to obtain quantitative measures of manufacturers' sales expectations.[15] We adopt a strategy which enables us to analyze the buffer

[13] It may be objected that the buffer stock model is not appropriate in those cases in which the scale of production is determined on the basis of specific orders for finished commodities; but Stanback may well have been justified in concluding that finished goods made to specific order are shipped almost upon completion so that variations in the size of this investory category make a negligible contribution to fluctuations in total finished goods inventory [**19**, p. 89]. Abramovitz has emphasized that a test of the buffer stock model is "plagued by lack of an objective standard by which to judge a surplus or deficit of stocks." He rejects the special case of the theory in which it is assumed that $\alpha = 0$, that manufacturers attempt to keep a constant ratio of inventories to sales [**1**, p. 152].

[14] Observe that an upper bound to actual sales is provided by the restraint that inventories cannot be negative.

[15] When data relating to expectations are accumulated, their validity may still be open to question. Albert G. Hart has made a gallant attempt to "reconstitute" Railroad Shippers Forecast data for the interwar period into a revised series more in conformity with preconceived concepts as to how expectations should behave [**7**].

stock inventory model without relying upon any attempt at measuring actual sales expectations. We look for the impact of errors in forecasting upon measured inventory and sales data. We shall find that the need to consider sales expectations does not only present difficulties in the analysis of inventories; it is also rewarding in that something may be inferred concerning the nature of sales expectations as well as desired inventories from observing actual sales and inventory behavior.

One possible strategy would be to adopt the assumption of static expectations, to let $\hat{X}_t = X_{t-1}$. But we will consider a more general hypothesis concerning the nature of expectations. After all, the assumption of naive expectations is an insult to the entrepreneur; surely he can do better than this. On the other hand, to assume perfect expectations is to attribute to him the power of the soothsayer.[16] John Maynard Keynes makes a provocative suggestion that may help us. He states [**9**, p. 51]:

> . . . it is sensible for producers to base their expectations on the assumption that the most recently realized results will continue except in so far as there are definite reasons for expecting a change.

If the firm's adjustment of the simple, naive projection based on definite information is in the right direction, the level of sales actually expected would fall between the two extremes of static and perfect forecasting. More precisely, we may hypothesize

$$\hat{X}_t = \varrho X_{t-1} + (1-\varrho)X_t\,, \qquad 0 \leqslant \varrho \leqslant 1\,. \tag{3.5}$$

If $\varrho = 1$ we have the extreme of static expectation; $\varrho = 0$ corresponds to the opposite case in which there is no systematic tendency either to over or to underestimate the actual change in sales.[17] Contrariwise, $\varrho < 0$ would imply that the actual change in sales is generally overestimated.

But how can one check the validity of such a reconstitution? One check is a pragmatic one: is the devised series useful in prediction? In connection with sales expectations, one appropriate test concerns the usefulness of the series in describing inventory behavior. The application of this test to a "sales expectations" series I devised from the same data utilized by Hart, but for the postwar period, gave a negative answer; the procedure described in this paper for obtaining a surrogative measure of expectations provides a closer prediction of actual inventory behavior.

[16] A perfect forecasting record may be achieved by a firm willing to push goods by advertising, salesmanship, or price cutting when sales lag behind the predicted quantity and, conversely, to refuse to sell additional goods once the forecast of sales has been fulfilled; obviously, such behavior is at the expense of profits. A basic assumption underlying the buffer stock type of model is that firms respond passively to changes in demand in the sense that they do not adjust either price or sales policy to short run changes in demand.

[17] Edwin S. Mills based an econometric investigation of inventory behavior on the assumption that sales anticipations deviate from actual developments by a random variable with zero mean [**17**].

The hypothesis expressed in equation (3.5) may be investigated by studying data on actual anticipations.[18] But this is by no means the only approach. Let us substitute the expression for expectations, (3.5), into equation (3.4), the flexible accelerator buffer stock inventory model. This yields

$$(3.6)\quad I_t = \delta\alpha + (\delta\beta + 1)\varrho X_{t-1} + (\delta\beta + 1)(1 - \varrho)X_t - X_t + (1 - \delta)I_{t-1}\,.$$

The estimates of the parameters of this model were obtained by applying least squares after it had been reformulated in the form

$$(3.7)\qquad I_t = \delta\alpha + \delta\beta X_t + (\delta\beta + 1)\,\varrho(X_{t-1} - X_t) + (1 - \delta)I_{t-1} + \varepsilon_t\,.$$

It is clear that unambiguous estimates of all parameters of the model may be obtained by unscrambling the coefficients obtained from the regression.

Estimates of the parameters of equation (3.7) are presented in Table II. It is to be observed that all point estimates of the reaction coefficients, the δ's, fall within the correct range; they are slightly smaller than the estimates obtained for stocks of purchased materials and goods in process. The estimated marginal desired inventory coefficients, all of correct sign, are larger for finished goods than for stocks in every case.

The estimates of the ϱ's, the anticipation coefficients, all lie between zero and unity, implying that firms in manufacturing typically anticipate a sizable share of the actual change in sales volume.[19] Even for nondurables, the estimate of 0.1685 suggests that on the average more than four-fifths of the actual change in sales is generally anticipated. But an alternative interpretation of the evidence is possible, an interpretation suggested to me by James Henderson and by Arthur Okun. Sales anticipations held at the beginning of the quarter may not be so unbiased; rather, a degree of flexibility in production scheduling may permit a partial modification of

[18] In a recently published study Theil reports on an extensive study of the prediction record of a number of forecasts made in several different countries. His investigation suggests a general tendency for anticipations to understate actual changes [**20**, Ch. V]. Hypothesis (3.5) was formulated and investigated independently of Theil's study.

[19] Since the calculations are based on aggregative data rather than figures for individual firms, the resulting estimates might result from a mixture of companies that make naive projections and other firms that typically overestimate the actual change in sales. Even if data on individual firms were available, a value of ϱ close to unity would have to be interpreted as implying that there is no systematic tendency generally to over or underestimate actual changes in sales, a lack of bias in forecasting that is not necessarily indicative of precision. Nevertheless, the results are of considerable interest; for one thing, there is no hint of any tendency for expectations to be "regressive" in the sense of having a general tendency to swing back in the opposite direction from recent changes, a characteristic of the questionable, unprocessed Railroad Shippers Forecast expectations data. Cf. Hart [7] for a discussion of an attempt to correct the raw railroad forecast data for its systematic, regressive tendency.

the original production plan when the actual change in sales is underestimated. Okun suggests that flexibility implies that actual stocks are related to the planned level by a coefficient of inflexibility in terms of the following modification of equation (3.4):

$$I_t = I_t^p + \lambda(\hat{X}_t - X_t)\,. \tag{3.4'}$$

If production plans are partially flexible, $1 > \lambda > 0$, an excess of actual sales over the anticipated level will lead to only a partial discrepancy between actual and planned end-period inventories. If production plans are completely inflexible, $\lambda = 1$, we have the special case already investigated. If, on the other hand, $\lambda = 0$, we would have the opposite extreme of complete flexibility that would be possible only if production were instantaneous rather than involving time. If one proceeds in the same way as before, one eventually derives from (3.4′) a revision of equation (3.7):

$$I_t = \delta\alpha + \delta\beta X_t - (\delta\beta + \lambda)\,\varrho(X_t - X_{t-1}) + (1-\delta)\,I_{t-1}\,. \tag{3.7'}$$

Now this equation is "underidentified" in the sense that we cannot unscramble the least squares coefficients in order to estimate ϱ and λ, although we still obtain precisely the same estimates of β, δ, and α as before. While this means that the effects of errors of anticipations cannot be segregated from production inflexibility empirically, Arthur Okun suggests that reasonable assumptions concerning the value of the coefficient of inflexibility still imply a quite high value of ϱ. Suppose, for example, that $\lambda = {}^1/_2$, presumedly a low value for a three month planning period. Then for total manufacturing the value of ϱ computed under this assumption is only 0.2427 rather than the figure of 0.1521 obtained under the assumption of complete inflexibility. A similar insensitivity holds for the other regressions. Although the assumption of complete inflexibility may imply an erroneously low value of ϱ, manufacturers still appear to anticipate on the average a large portion of changes in sales volume even when a considerable degree of production flexibility is assumed.

4. TOTAL INVENTORY BEHAVIOR: A BREAKDOWN BY INDUSTRY

Evidence concerning the behavior of stocks of purchased materials and goods in process was presented in Section 2; finished goods inventories were examined in Section 3. But both studies were based on a high level of aggregation. Ideally, of course, one would like to utilize a cross-section approach relying on data for individual firms. Failing this, a disaggregation of the totals, at least to an industry level, is most useful. Such a breakdown is possible with existing data currently released by the Office of Business Economics, but only at a cost. In return for the greater detail of data as to industry, it is necessary to sacrifice the classification of inventory by stage of fabrication.

In this section we report on an investigation based on an industry breakdown for five component durable goods industries. Let the variable H_t represent total inventory stocks held at the end of the period. Since S_{t+1} represents stocks of purchased material and goods in process in the hands of producers at the beginning of period $t+1$ while I_t stands for inventories of finished goods held at the end of period t, we have the identity

$$H_t \equiv S_{t+1} + I_t . \tag{4.1}$$

In view of this correspondence, it seems appropriate to add together the explanatory terms of equations (2.3) and (3.7), the expressions for stocks and for inventories, respectively. Since our measure of quantity, Q_t, cannot be derived without knowledge as to the change in finished goods inventory, this term in equation (2.4) must be approximated by sales. In this way we obtain as the equation explaining total inventories held by each industry

$$\begin{aligned} H_t = \delta\alpha + \delta\beta_1 X_t + (1-\delta)H_{t-1} - (\delta\beta_1 + 1)\varrho\Delta X_t + \delta\beta_2 U_{t+1} \\ + \delta\beta_3\left(\frac{\Delta p_{t+1}}{p_{t+1}}\right) + \varepsilon_t . \end{aligned} \tag{4.2}$$

It was possible to reprocess the data for total durable, total nondurable, and the total manufacturing sectors in addition to analyzing the new data for component industries. The statistics appear in Table III. A check on the effects of aggregating over stages of fabrication is provided by the total inventory estimates obtained for these sectors. Before turning to the individual industry estimates, let us test the consistency of these estimates for the aggregates with those obtained earlier. The comparison will lend support to the conjecture P. Darling advances "that aggregation may be pushed much further in inventory analysis than has heretofore been generally felt justified" [4, p. 958]. It will provide justification for the interpretation of the estimates obtained for individual industries where data broken down by stage of fabrication are, unfortunately, unavailable.

First of all, one would surmise that the total inventory reaction coefficient obtained for each sector would be a rough average of the sector's stock and inventory reaction coefficients obtained under the separate regressions utilizing the stage of fabrication breakdown. This indeed proves to be the case, as may be seen by comparing the data in Tables I and II with the figures presented here. Again, remembering that sales are now being utilized as a proxy for output, we should find the total marginal desired inventory coefficient to be roughly the sum of the coefficient for purchased materials and goods in process stocks and that for finished goods inventory; this indeed holds, the approximation being particularly good for total manufacturing and total durables. Also, for total manufacturing and total durables, if not for the nondurables, the unfilled orders coefficient is remarkably stable. The estimates of the coefficient of anticipations obtained when the stage-of-fabrication breakdown is neglected quite closely approximate those obtained with the finished goods regression; even the largest discrepancy, that for total manufacturing, is only a contrast of 0.13 versus 0.15 in estimating the anticipations coefficient. Only the coefficient of the rate of change in prices of purchased materials and goods in process shows a marked sensitivity to the level of aggregation; we find them larger in absolute value and all negative, implying that total inventories are smaller when prices of purchased materials are on the increase.

With the exception of a possible distortion of the role of prices, the estimates obtained when the data on finished goods inventory are combined with the figures for stocks of purchased materials and goods in process are remarkably consistent with the earlier estimates utilizing the breakdown. It seems quite reasonable to conjecture, then, that the estimates we present for the five component durable goods industries do not differ greatly from those that would have been obtained with separate regressions if stage of fabrication data could be utilized.

Let us turn to the data for the five component durable goods industries. It is to be observed that all reaction coefficients are of the correct sign and of reasonable magnitude. The point estimate of the reaction coefficient for the transportation equipment industry is largest, implying that firms in this industry attempt the most rapid adjustment of stocks to the desired level; firms in primary metals appear to be the slowest. Observe that the total durable reaction coefficient is larger than that for any component industry rather than a rough average of the estimates for different industries; this suggests that at least some distortion may be involved when data limitations require aggregating over component industries in order to work with data for a major sector of the economy.

All marginal desired inventory coefficients appear to be of correct sign. The considerable range in the value of these coefficients may be partially due to sampling errors as well as to interindustry differences in the extent to which the optimal level of inventories is actually related to the volume of sales. The point estimate is lowest for machinery; the largest coefficient, that for primary metal, is almost ten times the figure for machinery. The figure obtained from the aggregate regression for total durables was about three times the smaller figure.

The coefficients of anticipations are all positive and less than unity. This suggests that firms are successful in attempting to adjust the simple naive projection of sales in the direction of actual developments. They do not overshoot, but rather tend to underestimate the actual change in sales by a surprisingly small amount. In the transportation equipment industry a particularly low bias in forecasting changes in sales is suggested by the point estimate. But even in stone, clay, and glass—the least accurate industry considered—almost four-fifths of the actual change is anticipated on the average. The regressions for individual industries confirm the conclusion, based on the data for durable and nondurable totals, that manufacturers tend to anticipate both the direction and magnitude of changes in sales volume.

In sum, the parameter estimates obtained utilizing the industry data are reasonably consistent with the values one would expect on the basis of the data that are aggregated over industries but broken down by stage of fabrication. While the estimates are clearly inconsistent with the simple, naive accelerator hypothesis already rejected by Abramovitz, they are in conformity with more complicated versions of the accelerator which incorporate the flexible principle of Goodwin and the buffer stock concept of Lundberg and Metzler.

5. SURPLUS INVENTORIES

Discrepancies between the desired or equilibrium level of inventories and the actual size of stocks are explained by the complicated version of the acceleration principle developed in this paper, a model incorporating the flexible accelerator principle of Goodwin and the buffer stock concept of Lundberg and Metzler. Estimates of the parameters of the model were provided in Table III. In this section these estimates of the model's parameters are utilized to construct a series for the surplus inventory holdings of five durable goods industries for the years 1948 to 1955.[20]

The equilibrium level of inventories is defined as that level which entrepreneurs would work to obtain on the basis of the current level of sales and the backlog of unfilled orders if they were not disturbed by dynamic factors.[21]

[20] An alternative and equally feasible approach would be to discuss discrepancies between the observed and the desired inventory to sales ratio. If both sides of equation (4.2) are divided by X_t, an expression for the observed inventory-output ratio is obtained. If equation (5.1) below is also divided by X_t, one obtains an equation explaining the desired inventory to sales ratio in terms of both sales volume and the backlog of unfilled orders.

[21] A periodic survey currently conducted by the Office of Business Economics, Department of Commerce, inquires as to whether the firms' current inventory position is above, below, or approximately equal to the volume of stocks that the firm would like to hold on the basis of current sales and the backlog of unfilled orders, but a figure as to the size of the surplus or deficit is not requested. Lawrence Klein utilized residuals from an equation predicting actual inventories as a measure of undesired stocks in one of his econometric investigations [11, p. 102].

The equation for estimating the desired level of inventories is obtained from (4.2) by eliminating the dynamic disturbances of price changes and errors in anticipating future sales, by setting $\Delta p_{t+1}/p_t = \Delta X_t = 0$. Then,

$$H_t^d = \alpha + \beta_1 X_t + \beta_2 U_t \,. \tag{5.1}$$

Here are the estimates of the parameters of the equilibrium level of inventory equation for each of the five durable goods industries:

	α	β_1	β_2
Stone, Clay, and Glass	102.7	.4068	—
Primary Metal	3119.1	1.1390	.3231
Transportation Equipment	841.7	.2617	.1009
Machinery	2514.9	.1161	.1983
Other Durable	168.5	.6278	—

If equation (5.1) is subtracted from (4.2) one obtains an equation for estimating surplus inventories:

$$H_t - H^d = (\delta - 1)\alpha + (\delta - 1)\beta_1 X_t + (1 - \delta)H_{t-1} - (\delta\beta_1 + 1)\varrho\Delta X_t + (\delta - 1)\beta_2 U_{t+1} + \delta\beta_3 \frac{\Delta p_{t+1}}{p_{t+1}} \,. \tag{5.2}$$

The estimates of the parameters of this equation for the five durable goods industries appear in Table IV.

TABLE IV

COEFFICIENTS OF SURPLUS INVENTORY EQUATIONS

	Stone, Clay, and Glass	Primary Metal	Transportation Equipment	Machinery	Other Durables
$(\delta - 1)\alpha$	75.3—	2946.3	575.7—	1762.9—	135.7—
$(\delta - 1)\beta_1$	.2983—	1.0759—	.1790—	.0814—	.5057—
$(\delta - 1)\beta_2$	—	.3052—	.0690—	.1390—	—
$\delta\beta_3$	.0025	.0355—	.0057	.0288	—
$-(\delta\beta_1 + 1)\varrho$	.2343—	.0432—	.0306—	.0697—	.1347—
$(1 - \delta)$	.7333	.9446	.6840	.7010	.8055

These last coefficients were applied to the same data utilized in the original regression in order to obtain time series representing surplus inventory holdings for each of the durable goods industries. For each quarter the surplus holdings of the five industries were summed so as to provide an estimate of total surplus inventory holdings for the durable manufacturing

TABLE V

SURPLUS INVENTORIES IN DURABLE GOODS INDUSTRIES

(Millions of 1947 Dollars)

Years and Quarters	Total Durable		Surplus Inventories by Industry				
	Actual Inventories	Surplus Inventories	Stone, Clay, and Glass	Primary Metal	Transportation Equipment	Machinery	Miscellaneous Nondurables
48–1	13,739		37–		258	180	
2	13,796	836–	26–	502–	219	140	667–
3	13,826	611–	22–	437–	193	268	613–
4	13,835	876–	28–	548–	61	271	632–
49–1	13,900	748	22	132–	162	212	484
2	13,517	1,633	17	494	198	294	630
3	13,001	1,604	15	772	118	207	492
4	12,591	1,959	4–	1,330	131	51	451
50–1	12,463	184–	59–	197	30–	31	323–
2	12,765	1,688–	108–	575–	215–	176–	614–
3	12,878	3,021–	138–	1,077–	474–	434–	898–
4	13,656	3,322–	133–	1,218–	524–	567–	880–
51–1	14,102	3,540–	101–	1,149–	629–	844–	817–
2	15,475	4,018–	79–	1,735–	673–	1,057–	474–
3	17,023	2,717–	3–	1,613–	428–	831–	158
4	17,708	2,254–	28	1,402–	321–	694–	135
52–1	18,312	1,549–	18	969–	250–	564–	216
2	18,319	640–	54	2–	286–	446–	40
3	18,381	1,143–	17	173–	316–	376–	295–
4	18,719	2,674–	16–	1,139–	595–	469–	455–
53–1	19,122	2,091–	32–	937–	549–	320–	253–
2	19,741	1,788–	10–	995–	431–	149–	203–
3	19,865	534–	3–	594–	149–	124	88
4	19,461	1,391	14	488	206	384	299
54–1	18,893	1,872	24	938	205	556	149
2	18,274	1,914	16–	985	186	604	155
3	17,724	1,535	41–	853	151	504	68
4	17,560	1,047	51–	748	104	402	156–
55–1	17,493	187–	91–	40	33–	232	335–
2	17,799	967–	106–	432–	10	115	554–
3			93–	857–	95	116	

sector of the economy. The data appear in Table V together with a constant dollar series of actual durable manufacturing inventories. The provisional character of the derived series of surplus inventories must be emphasized; their validity depends upon the accuracy of our specification of the equation explaining actual inventory behavior. They are obviously subject to sampling error. While the evidence presented should be considered as most tentative, it nevertheless provides an interesting interpretation of the behavior of durable goods inventories during a period of military mobilization. At the end of 1949 durable manufacturing firms held considerable excess stocks; almost one-sixth of their total inventory holdings were surplus. Three months later stocks were deficient. They remained below the equilibrium level until the end of 1953. From midyear 1950 to midyear 1951 durable goods manufacturing firms accumulated some two and one-half billion dollars of inventories, measured in 1947 prices. But this dramatic rate of inventory investment was not sufficient to prevent the deficiency of stocks from enlarging considerably. Indeed, actual inventory accumulation during the first year of the Korean emergency was only slightly more than one-half the change in the desired level of stocks. Although the deficiency in inventory holdings reached its peak at the end of June, 1951, the additional stocks necessary to eliminate the gap between desired and actual stocks were not accumulated until two more years had elapsed.

It may well be that inventory investment would have been much larger during the period of military mobilization except for two basic factors. First of all, firms in manufacturing follow a flexible inventory policy, attempting only a partial adjustment of actual inventories to the desired level during each production period. Second, our estimates of the coefficients of anticipations all imply a tendency for manufacturers to underestimate actual increases in sales. It is tempting to hypothesize that during the Korean crisis these two factors, by reducing actual inventory investment, served to limit effective demand during a period of inflation. This would imply that inaccurate expectations and a flexible inventory policy may at times serve to stabilize the economy. This is a conjecture concerning the behavior of the economy for alternative, hypothetical values of the parameters of the equation determining inventory behavior. But it must be observed that if the parameters of the equations explaining inventory investment in certain sectors of the economy had been different, the level of sales and possibly the backlog of unfilled orders might well have been affected; consequently, inventory behavior would have been different from that implied by considering the actual level of sales. The interdependent nature of the economy means that considerable difficulty is involved in appraising the effects of alternative inventory practices. A paper on theoretical problems related to the present article will appear in a future issue of this journal.

The Cowles Foundation for Research in Economics, Yale University

REFERENCES

[1] ABRAMOVITZ, MOSES: *Inventories and Business Cycles, With Special Reference to Manufacturers' Inventories*, New York: National Bureau of Economic Research, 1950.

[2] CHENERY, HOLLIS B.: "Overcapacity and the Acceleration Principle," *Econometrica*, Vol. 20, January, 1952, pp. 1–28.

[3] COBREN, GEORGE M.: "The Nonfarm Business Inventory Component," in *Conference on Research in Income and Wealth* (Studies in Income and Wealth, Vol. 12), New York: National Bureau of Economic Research, 1950, pp. 379–408.

[4] DARLING, P. G.: Manufacturers' Inventory Investment, 1947–58," *American Economic Review*, XLIX (December, 1959), pp. 950–963.

[5] DURBIN, J., AND G. S. WATSON: "Testing for Serial Correlation in Least Squares Regression, II," *Biometrika*, Vol. 38, 1951, pp. 159–178.

[6] GOODWIN, RICHARD M.: "Secular and Cyclical Aspects of the Multiplier and Accelerator," in *Income, Employment and Public Policy: Essays in Honor of Alvin H. Hansen*, New York: Norton and Co., 1948.

[7] HART, ALBERT G.: "Quantitative Evidence for the Interwar Period of the Course of Business Expectations: a Revaluation of the Railroad Shippers Forecast," in *The Quality and Economic Significance of Anticipations Data*, Princeton: Princeton University Press, 1960, pp. 205–39.

[8] HURWICZ, L.: "Least Squares Bias in Time Series," in T. C. Koopmans, ed., *Statistical Inference in Dynamic Economic Models*, Cowles Commission Monograph Number 10, New York: John Wiley, 1950, pp. 365 ff.

[9] KEYNES, JOHN M.: *The General Theory of Employment, Interest and Money*, New York: Harcourt, Brace, 1936.

[10] KLEIN, LAWRENCE: *Econometrics*, Evanston: Row, Peterson, 1953.

[11] ———: *Economic Fluctuations in the United States*, 1921–41, Cowles Commission Monograph Number 11, New York: John Wiley, 1950.

[12] LOVELL, MICHAEL C.: *Inventories and Stability*: *An Interindustry Analysis*, Unpublished Ph.D. Dissertation, Harvard University, March, 1959.

[13] LUNDBERG, ERIC: *Studies in the Theory of Economic Expansion*, London: P. S. King and Son, 1937.

[14] MACK, RUTH P.: "Characteristics of Inventory Investment: The Aggregate and its Parts," in *Problems of Capital Formation*: *Concepts, Measurement, and Controlling Factors* (Studies in Income and Wealth, Vol. 19, National Bureau of Economic Research), Princeton: Princeton University Press, 1957, pp. 471–486.

[15] MANNE, H. B., AND A. WALD: "On the Statistical Treatment of Linear Stochastic Difference Equations, " *Econometrica*, Vol. 11, 1943, pp. 173 ff.

[16] METZLER, LLOYD A.: "The Nature and Stability of Inventory Cycles," *Review of Economics and Statistics*, Vol. III, August, 1941, pp. 113–129.

[17] MILLS, EDWIN S.: "The Theory of Inventory Decisions," *Econometrica*, 25 (April, 1957), pp. 222–239.

[18] MODIGLIANI, FRANCO: "Business Reasons for Holding Inventories and Their Macro-Economic Implications," *Problems of Capital Formation* . . . (see [14]), pp. 499–506.

[19] STANBACK, THOMAS M., JR.: "Cyclical Behavior of Manufacturers' Inventories Since 1945," *Proceedings, Business and Economic Statistics Section, American Statistical Association*, Washington, 1957, pp. 87–95.

[20] THEIL, H.: *Economic Forecasts and Public Policy*, Amsterdam: North-Holland Publishing Co., 1958.

[21] WOLD, HERMAN: *Demand Analysis*: *A Study in Econometrics*, New York: John Wiley, 1953.

MEASURES OF INVENTORY CONDITIONS

BY

NESTOR E. TERLECKYJ ASSISTED BY ALFRED TELLA
National Industrial Conference Board

PREFACE

The following technical paper, "Measures of Inventory Conditions," contains as a summary of its analysis two estimating equations, one for the 3-month periods and the other for 6-month periods, which express net changes in trade and manufacturing inventories over a short period as a function of ratios of inventories, new orders, and unfilled orders to sales, prevailing at the beginning of the period. These equations were fitted to seasonally adjusted quarterly data from March 1948 through September 1959. It appears that these relationships have enjoyed a considerable degree of stability during the period on which they were based, judging from the consistency of fit of the estimating equations themselves and, perhaps more suggestively, from the fact that very similar equations were obtained for two shorter periods, 1948–54 and 1948–57, by Thomas M. Stanback. In his paper* Professor Stanback refitted the 6-months' equation for these periods, using the same data which went into the calculation for the longer period.

By now 2 years have passed since the end of the period to which the equations were fitted, and it may be appropriate to inquire what their performance has been since that time. The record of the past 2 years suggests three general observations which apply to both the 3-month and the 6-month estimating equations:

(1) The cyclical drift of the calculated inventory changes corresponded to that of the actual series. The overall correspondence of the turning points, subject to reservations mentioned below, appears to have been about as good as that which prevailed over the period 1948–59. It appears, however, that the calculated changes have understated the amplitude of fluctuations that occurred in the actual series.

(2) There appears to have been no divergence in the *level* of the two series. The deviations between the actual and calculated inventory changes have occurred in both directions, and during the past 2 years the actual and the calculated curves have crossed each other three times.

(3) The overall correspondence between the actual net percentage changes in business inventories and the changes calculated from the equations has not been as close in the last 2 years as over the longer period to which the equations have been fitted. In the second half of 1959, during the steel strike, the equations suggested considerable accumulation of inventories while the actual series shows almost no change. In the first half of 1960 the rates of inventory accumulation suggested by the equations were considerably below the rates that actually prevailed. Then, during the 1960–61 recession the formulas indicated small amounts of accumulation (and only in one instance a very small rate of liquidation) while in actuality significant liquidation

* "A Critique of Inventory Forecasting Approaches," Proceedings of 1960 Meetings, American Statistical Association.

occurred. According to the latest results, based on preliminary data for September 1961, the two series have moved closer together, and the calculated rate of inventory accumulation is again running below the actual.

It could well be that the generally greater deviations between the actual inventory changes and the changes indicated by the estimating equations in the past 2 years represent a passing effect of a large disturbance introduced into the underlying relationship by the 1959 steel strike. In that case the basic patterns should reassert themselves in the near future. On the other hand if these deviations remain as large as in the past 2 years or increase in the course of the next several quarters, a case could be made for suspecting a structural change in the underlying relationships. A comprehensive review of the choice and relative weights of determinants of inventory change would then be indicated.

MEASURES OF INVENTORY CONDITIONS

By Nestor E. Terleckyj

Assisted by Alfred Tella

NATIONAL INDUSTRIAL CONFERENCE BOARD, INC.

FOREWORD

While long-term economic trends are shaped by the production and sale of end-products, fluctuations in business inventories have played an important and often dominant role in short-term business movements of the past fifteen years. It is a rare business enterprise that is effectively sheltered from the strong ebb and flow of inventory cycles. Accordingly, economists and business managers have directed increasing attention to estimating the probable course of inventory demand, as a guide to general business conditions and conditions in their own industries.

For much of the period since the end of World War II, inventory policy took shape in an environment of scarce resources, low interest rates and rising commodity prices. As long as these conditions remained fundamentally unchanged, relatively simple tools of measurement appear to have explained inventory trends fairly satisfactorily. More recently, however, as general supply conditions have eased, as prices have stabilized, and as interest rates have risen, the behavior of inventory demand has evidently grown more complex.

This paper presents a historical and an analytical record of the behavior of inventories in the postwar years, together with a number of methods for studying the current and prospective course of inventories, both in aggregate and by industry. Many of the series developed here will continue to be reported on a current basis in *The Conference Board Business Record* and the *Weekly Desk Sheet.*

Albert T. Sommers, Director
Division of Business Analysis

THE MEASURES OF INVENTORY CONDITIONS

I. The Role of Inventories in the Business Cycle

The importance of inventories in the business cycle stems from their role as the volatile residual difference between two large aggregates: total supply and total end-product demand. While the production and sale of goods are continuous economic processes that must adjust to each other over the long term, rarely within any given time period is the total volume of goods produced precisely equaled by the volume of final sales. The stock of goods held in inventories thus increases or declines over any given period, depending on whether production exceeds or falls short of final sales.

This inventory discrepancy exhibits a definite and often dramatic cyclical pattern. There is a strong cycle in the *rate of accumulation* of business inventories, which is translated into fluctuations in the stock of inventories, and thence into fluctuations in the relationship of the stock to general business activity.[1]

In the late Fifties, the value of goods held in business inventories was in the neighborhood of $100 billion. The annual value of goods output was more than twice that amount, and the annual value of goods and services production (gross national product) was four to five times as large. Of the $100 billion of goods held in nonfarm inventories, about $90 billion was held by firms in manufacturing and trade, and the remaining $10 billion by the construction, utilities, and other industries. Of total manufacturing and trade stocks, manufacturers of durable goods accounted for some $30 billion; manufacturers of nondurable goods for over $20 billion; wholesalers for about $12 billion; and retailers for some $25 billion.

Chart 1 illustrates the postwar fluctuations in gross national product, in the stocks of manufacturing and trade inventories, and in the rate of inventory accumulation. As may be readily observed, fluctuations in the stock of inventories have matched the movements in the gross national product quite closely, although with a significant lag. On the other hand, the cycles in the *net change* of business inventories, shown in the lower section of the chart, appear to have preceded the movements of GNP.

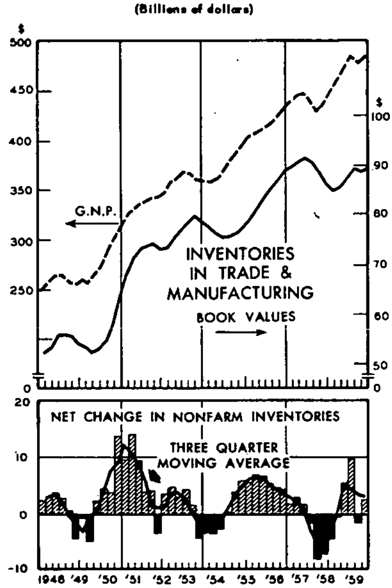
The net change in business inventories is itself a part of GNP. A net increase in inventories between the beginning and the end of an accounting period represents that part of the period's gross product used to augment the stocks of goods on hand. Conversely, a net decrease in inventories – that is, a withdrawal from stocks – indicates final demand has been above the level of output, and the inventory change must be subtracted from the demand estimates in order to arrive at the value of output.

Just as the absolute *change* in inventories (as distinguished from the stock of inventory) is on the same "flow" basis as GNP itself, so *changes in the rate* of

Chart 1

Gross national product, business inventories, and net changes in nonfarm inventories; quarterly 1948-1959[a]

(Billions of dollars)



[a]Seasonally adjusted data; GNP and net change in nonfarm inventories at annual rates

Sources: Department of Commerce; The Conference Board

[1]The movement of farm inventories, which is largely independent of general economic conditions, is omitted from the present analysis

inventory accumulation or liquidation are comparable to changes in the *rate* of total output. Such a comparison amounts to an appraisal of the contribution made by fluctuations in the rate of inventory formation (which may be positive or negative) to fluctuations in general business activity. This contribution has differed substantially over time, depending on the stage of the cycle, but has been similar for comparable stages of the postwar cycles.

The variation in relative importance of inventory change over the cycle is owing to a characteristic lead of changes in the rate of inventory accumulation over the corresponding fluctuations in gross national product. As shown in Chart 1, the *rate* of inventory accumulation (net change in business inventories) has typically reached its peak and begun to decline within the first year and a half of general business expansion. This decline has tended to continue throughout the later stages of the boom, and throughout most of the recession, until the bottom is reached about one quarter prior to the upturn in GNP. The rate of accumulation of inventories then begins to rise again. In its ups and downs, the rate of inventory change tends to cross the line between positive and negative values in general correspondence to the turning points in GNP, but with a slight lag.

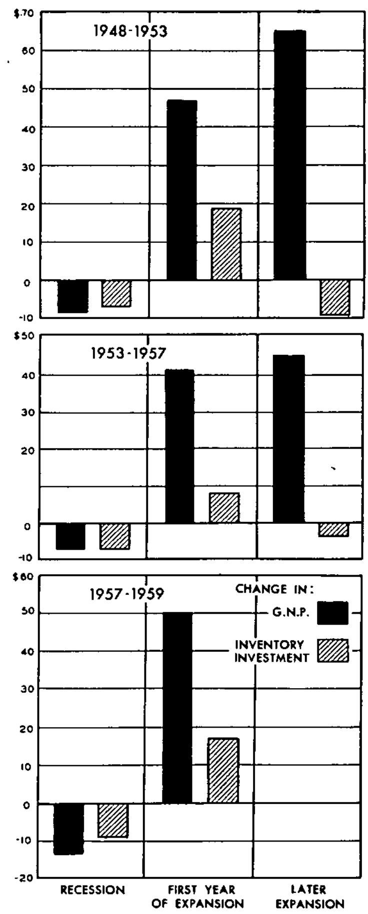
The dissimilarity in the direction of movements of gross national product and the rate of inventory accumulation, which has characterized the postwar experience, occurs chiefly in the late stages of business expansions. During recessions, both GNP and the rate of inventory accumulation decline; during the first year of expansion, both advance. During the later phases of expansion, however, the rate of inventory build-up tends to enter on its cyclical decline while GNP is still expanding.

As Chart 2 shows, the recession declines in inventory demand have been almost as great as the aggregate declines in gross national product, leaving only a relatively minor share of the decline to all the other sectors of the economy taken together. This suggests the reason why postwar business downturns have often been called "inventory recessions." One can also observe from the chart that the contribution of inventory

Chart 2

Changes in GNP and in the rate of nonfarm inventory investment during recession and expansion phases of postwar cycles▲

(Billions of dollars)



▲Based on seasonally adjusted quarterly data. Recession is the period from cyclical peak to trough and expansion the period from trough to peak. The postwar peaks occurred in the following calendar quarters: IV 1948, II 1953, and III 1957; the troughs in IV 1949, III 1954, and II 1958

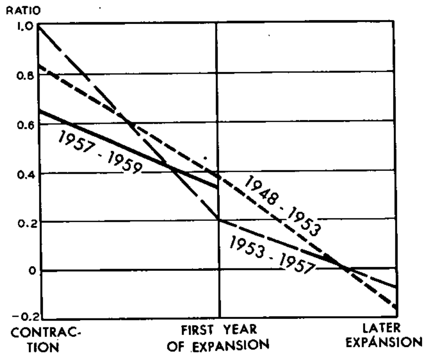
Sources: Department of Commerce; National Bureau of Economic Research; The Conference Board demand has been much more modest in expansions than in recessions, and has been confined to the first year of expansion.[1] The momentum of postwar expansions has been provided by vigorous cyclical growth in end-product demand. In the first year of the postwar expansions, the contribution of inventory build-up to increases in total output was positive, amounting to roughly one third of the total; in the late phases of expansion, the net increases in inventory demand were negative, the entire expansion being accounted for by increases in final sales.

The relative contribution of fluctuations in the rate of inventory accumulation to general economic swings in the various phases of the postwar cycles is summarized in Chart 3, which shows the ratios of the changes in inventory accumulation to changes in gross national product in three phases of the cycle. The chart indicates that, in recessions, inventory change has accounted for between two thirds and all of the decline. In the first year of postwar expansions, its contribution ranged from 20% to 40% of the upward movement. In the later phases of the upturns, change in inventory demand was negative and acted as a brake on expansion.

Inventory change has thus played a major and complex role in the postwar business cycle, influencing both the timing of cycle turning points and the ensuing rates of expansion and contraction.

Chart 3

Ratio of change in the rate of inventory investment to change in GNP during various stages of the postwar cycles▲



▲Based on seasonally adjusted quarterly data
Sources: Department of Commerce; National Bureau of Economic Research; The Conference Board

II. An Introduction to Inventory Statistics

The available general inventory statistics differ with respect to prices at which the inventories are valued, coverage, detail, timeliness, and frequency of reporting.

Goods held in inventories may be expressed in terms of "book value," that is, in the prices in which they are currently carried on the firms' books. This valuation depends on the prevailing combination of inventory accounting practices, such as LIFO (last in first out) and FIFO (first in first out) methods. FIFO bookkeeping is still the most prevalent method, and the available "book value" inventory data tend to reflect price levels of the recent past, rather than the present. Moreover, the change in book-value inventories will reflect changes in prices; in a period of rising prices, for example, a unit removed from inventory will be valued at a lower figure than the unit purchased to replace it, and aggregate book value will increase although the physical volume remains unchanged.

Alternatively, inventories may be expressed in current prices, that is, prices prevailing at the time of reporting (sometimes called "replacement prices") or in LIFO prices. The effect of LIFO accounting is to place *inventory change* on a "replacement price" basis, that is, the amount of change in the inventory level will be equal to the physical volume of change, valued at current prices. Nevertheless, the *total stock* of inventory, under LIFO, is still valued at historical rather than current prices.

Finally, inventories may be expressed in "constant" prices, that is, in prices prevailing in a selected standard period. Where the stock of inventories is expressed in "book value" or "replacement value," period-to-period changes in the value of the stock may reflect price changes, as well as changes in physical quantities; where the stock is expressed in constant prices, period-to-period changes in the stock reflect changes in physical quantities only.

Book values of inventories held by trade and manufacturing firms are reported in rich detail as of the

[1] "Contribution" here does not mean causation, but simply the share of the fluctuation in the total attributable to inventory demand

end of each month, in both unadjusted and seasonally adjusted form, by the United States Department of Commerce. These figures, which are obtained through a continuing sample survey, become available about five weeks after the date of reporting.

Changes in nonfarm inventories, valued in both current prices and constant prices, appear quarterly and annually as part of the Commerce Department's *National Income and Product* reports, with a lag of about seven to eight weeks after the end of each quarter. These *change* figures are developed from the book-value figures described above, but are converted to a LIFO basis by adding or subtracting an "inventory valuation adjustment." They are thus representative of the physical change, valued at prevailing prices.

Table I presents several statistical series on inventory change. In some years shown in the table, the estimates of inventory change differ considerably from each other. The difference between the figures in column 1 and those in column 2 arises from discrepancies in definitions of trade inventories, from differences in adjusting the sample results to bench-mark totals, and from different methods of seasonal adjustment (which affect the annual changes in some small degree). Some of the discrepancies arising from differences in definition will probably be reduced in the revised monthly

TABLE I

ANNUAL CHANGES IN BUSINESS INVENTORIES, 1948 – 1958

(Billions of dollars)

Year	Monthly Survey Data, Trade and Manufacturing, Book Values	National Income Accounts			
		Trade and Manufacturing, Book Values	Nonfarm Total		
			Book Values	Current Prices	1954 Prices
	(1)	(2)	(3)	(4)	(5)
1948	$4.9	$5.2	$5.5	$3.0	$3.0
1949	−3.6	−4.0	−4.5	−2.2	−2.6
1950	11.6	11.3	12.1	6.0	6.5
1951	10.4	9.6	10.6	9.1	9.0
1952	1.6	.9	1.0	2.1	2.2
1953	3.2	2.7	2.2	1.1	1.1
1954	−3.1	−1.8	−1.8	−2.1	−2.1
1955	6.2	7.5	7.4	5.5	5.4
1956	7.4	7.4	8.3	5.1	4.9
1957	1.6	2.5	3.0	1.2	1.1
1958	−5.5	−4.4	−4.4	−4.9	−4.4
Frequency of Reporting	Monthly	Annually	Annually	Quarterly	Quarterly

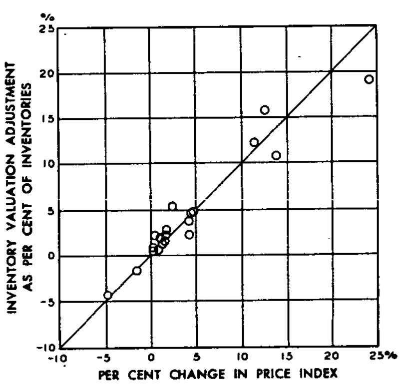
Source: U.S. Department of Commerce

figures on trade and manufacturing inventories for the period beginning with 1955, scheduled to be published by the Department of Commerce in 1960.

Column 2 differs from column 3 in coverage. While the series in column 2 includes changes in trade and manufacturing inventories only, the series in column 3 includes, in addition, the changes in inventories held in other nonfarm sectors such as mining, construction, and transportation. Column 4 differs from column 3 by an amount called "the inventory valuation adjustment" (IVA). Broadly speaking, this adjustment amounts to eliminating that portion of the change in book-value inventories that occurred because of changing prices, rather than changing physical volume of stocks. In effect, it represents the change in end-of-period inventories attributable to the charging of historical costs, rather than replacement costs, in inventory accounting. The IVA itself is a measure of the inventory profit

Chart 4

Relationship between changes in industrial whosesale price index and the inventory valuation adjustment,[a] annual data, 1939-1958



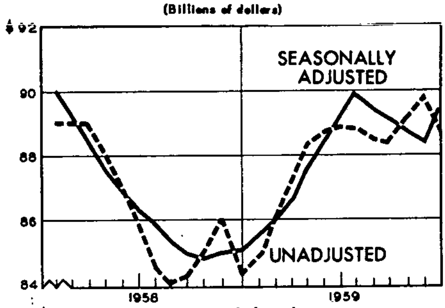
[a] Percentage change in industrial wholesale price during the year and inventory valuation adjustment as per cent of inventories (average of beginning and end of the year) for the same year. The sign of inventory valuation adjustment here is opposite to that conventionally given by the Department of Commerce

Sources: Department of Labor; Department of Commerce; The Conference Board

Chart 5

The seasonal pattern in trade and manufacturing inventories [a]

(Billions of dollars)



[a] Inventory book values at end of month

Source: Department of Commerce

or loss accruing to the holders of inventory as a result of the impact of changing prices[1] on book-value accounting.

The size of the inventory valuation adjustment depends on the rate of change in prices. While the procedures used by the Department of Commerce to estimate the adjustment are complex, the underlying principle is illustrated by the correlation of the percentage change in industrial wholesale prices during a given year with the corresponding inventory valuation adjustment, expressed as a percentage of the average level of inventories for the same year. This correlation is shown in Chart 4.

The figures in column 4 of Table I represent the *physical volume* of inventory change, valued at current prices, that is, at prices prevailing during the accounting period. The data shown in column 4 thus still reflect changing price levels in their valuation. In column 5, the inventory series is fully adjusted to constant prices by valuing each year's physical change in inventories at constant (1954) prices. This process of price deflation is done in considerable detail for purposes of the national accounts. However, the transition from current to constant dollars inventory change (column 4 to column 5) may be approximated by deflating the current-dollar change in nonfarm

[1] For a fuller explanation of the IVA, see United States Department of Commerce, "National Income," 1954, p. 59

inventories by the index of industrial wholesale prices (see Table II). This procedure eliminates almost the entire gap between the current dollar inventory changes and the officially reported changes in 1954 dollars.

As a final note on the statistics, seasonal influences on inventory holdings are considerable. The magnitude of the seasonal effect may be gauged from Chart 5, which shows, for a recent period, end-of-month book values of trade and manufacturing inventories both with and without seasonal adjustment. In recent years, for example, business inventories have been seasonally low in July and August by some $1.0 to $1.5 billions, and seasonally high in November by a similar amount. Seasonally adjusted inventory data are used throughout this report.

TABLE II

NET CHANGE IN NONFARM INVENTORIES IN CURRENT AND 1954 PRICES, 1939 – 1958

(Billions of dollars)

Year	Current Dollars	Current Dollars Deflated by Index of Industrial Wholesale Prices (1954 = 100)	1954 Dollars
1939	$ .3	$ .6	$ .6
1940	1.9	3.7	3.8
1941	4.0	7.1	7.6
1942	.7	1.2	1.6
1943	–.6	–1.0	–.5
1944	–.6	–1.0	–1.1
1945	–.6	–1.0	–1.6
1946	6.4	9.4	9.1
1947	1.3	1.6	1.4
1948	3.0	3.3	3.0
1949	–2.2	–2.5	–2.6
1950	6.0	6.5	6.5
1951	9.1	9.0	9.0
1952	2.1	2.1	2.2
1953	1.1	1.1	1.1
1954	–2.1	–2.1	–2.1
1955	5.5	5.4	5.4
1956	5.1	4.8	4.9
1957	1.2	1.1	1.1
1958	–4.9	–4.5	–4.4
1959	3.9	3.5	3.5

Sources: U. S. Department of Commerce, Bureau of Labor Statistics; The Conference Board

III. Secular and Structural Changes in Inventory Relationships

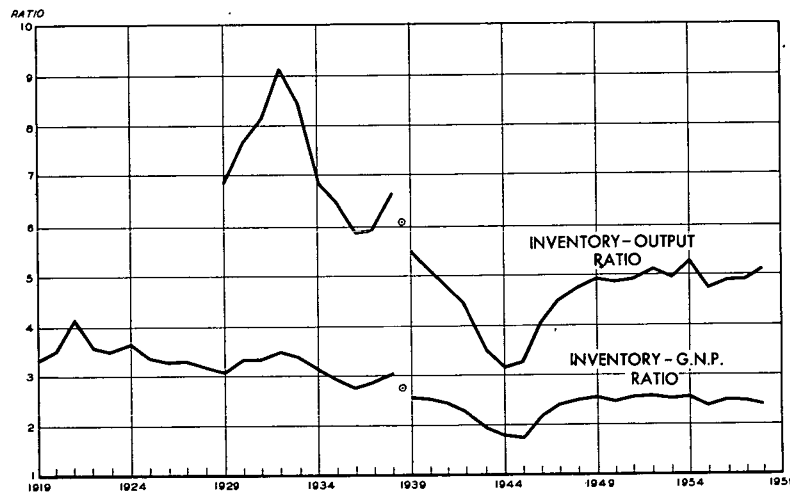
One of the most useful tools in an analysis of inventory conditions is the relationship between the level of inventories and the rate at which goods are currently produced or sold. This relationship, commonly expressed as a ratio, enables the observer of the inventory scene to judge whether, in light of past experience, inventories appear to be adequate, too high, or insufficient, relative to the current level of production or sales.

This judgment presupposes some notion of a longer-term "normal" relationship, abstracted from past experience, about which the short-term fluctuations occur. In practice, the normal relationship may be represented by a historical average or a trend value, adjusted, perhaps in an intuitive fashion, for various additional considerations.

Qualifications that attach to this procedure spring in part from the fact that goods are not homogeneous. The level of inventories required to support a given volume of production or sales is different for different types of business, and the relative weights of these types of business change over time. In addition, the prevailing practices controling the level of inventories continuously undergo change, as a result of changes in production technology, in techniques and policies of handling inventories, and a host of other factors. The ratio of inventories to output may thus be taken to reflect both a prevailing composition or "mix" of industries, and a current "technology" broadly defined to include organizational techniques and policies. This section is intended to provide some insight into the behavior of inventories, relative to the rate of flow of goods, over those relatively long periods which form the bases for calculations of "normal" in inventory analysis.

Chart 6

Ratios of business inventories to output of nonfarm goods and to GNP,▲ 1919-1958



⊙Change in definition of series

▲Ratios of trade and manufacturing inventories to average monthly GNP or output of nonfarm goods

Sources: Abramovitz, Moses, *Inventories and Business Cycles*, National Bureau of Economic Research, New York, 1950; Department of Commerce; The Conference Board

Behavior of Stock-Floor Ratios

Viewed in broad terms, the stock of inventories has been declining, relative to the volume of national economic activity, over considerable stretches of time. This tendency is reflected in a falling ratio of trade and manufacturing inventories to total national output, as shown in Chart 6 for the years 1919-1958. Throughout the 1920's and the 1930's the ratio fluctuated around a declining trend. It fell sharply again during World War II, then recovered and remained practically unchanged throughout most of the postwar period, at about the level reached in the years immediately preceding the war.

In so far as data are available for comparison, the ratio of business inventories to output of nonfarm goods followed very much the same trends. This suggests that the long-term decline in the level of inventories, relative to the level of economic activity, has reflected structural changes in production and marketing of goods, rather than merely an increased importance of services relative to goods.

One might expect that different types of inventory holdings, by stage of fabrication, are tied in different degrees to the level of output. Thus the technological connection with the rate of output should be stronger for materials and goods-in-process inventories than for the stocks of finished goods, and stronger for goods-in-process than for materials inventories.

The experience of World War II provides a striking illustration of this point. It suggests that in both durable and nondurable sectors of the economy, finished goods inventories (including both manufacturers' stocks and trade inventories) were flexible relative to output, and the goods-in-process stocks relatively inflexible. Although under the wartime pressures for speedy deliveries of finished goods and the shortages of materials, the inventory-output ratios declined for inventories at all the stages of fabrication, the magnitudes of declines were sufficiently different to suggest varying degrees of technological rigidity. The inventory-output ratios declined as follows from 1940 to 1944:

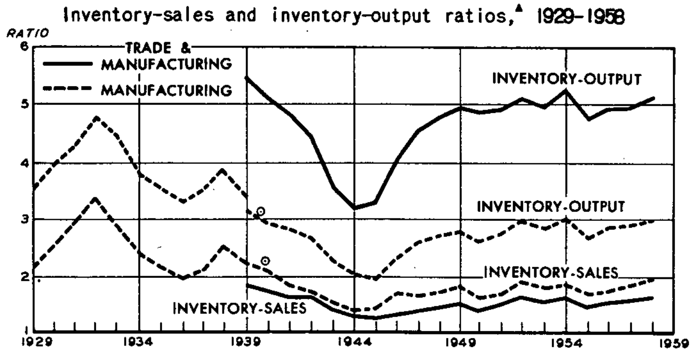
	Durable Goods	Nondurable Goods
Finished goods	- 71%	- 32%
Purchased materials	- 47%	- 5%
Goods in process	- 16%	- 5%

In the postwar period, when the disturbances were much less severe and offsetting influences were at work, the underlying technological relationships, though still in evidence, were not manifested as clearly as during the war.

The relation between the stock of goods and the rate of flow of goods, which is reflected in the ratio of inventories to output, also finds its expression in the inventory-sales ratio. These two alternative

Chart 7

Inventory-sales and inventory-output ratios,▲ 1929-1958



⊙Change in definition of series

▲Ratios of trade and manufacturing inventories to average monthly sales or output of nonfarm goods

Sources: Abramovitz, Moses, *Inventories and Business Cycles*, National Bureau of Economic Research, New York, 1950; Department of Commerce; The Conference Board ratios have differed in their absolute level and in their long-term trends. Because many goods are resold in successive stages of production and distribution before they reach final users, sales of goods have been several times higher than the value of goods produced.[1] The long-term movements of the two ratios have differed because in the long run the growth of sales has

[1] In 1957, trade and manufacturing sales were 3.1 times the value of nonfarm goods output; in 1939 they were 3.0 times as great

TABLE III

INVENTORY-SALES RATIOS IN TRADE AND MANUFACTURING, BY SECTOR, 1939-1959[a]

Year	Total Trade and Manufacturing	MANUFACTURING		WHOLESALE TRADE		RETAIL TRADE	
		Durable Goods	Nondurable Goods	Durable Goods	Nondurable Goods	Durable Goods	Nondurable Goods
1939	1.81	2.74	1.84	2.00	1.18	2.22	1.31
1940	1.75	2.32	1.85	1.67	1.11	2.09	1.30
1941	1.61	1.95	1.68	1.50	1.09	2.00	1.28
1942	1.61	1.83	1.62	1.50	1.04	3.00	1.35
1943	1.42	1.57	1.47	1.38	.87	2.50	1.26
1944	1.31	1.48	1.41	1.38	.82	1.83	1.15
1945	1.30	1.52	1.42	1.44	.83	1.77	1.04
1946	1.36	2.08	1.45	1.33	.80	1.39	1.08
1947	1.41	1.97	1.48	1.45	.81	1.52	1.19
1948	1.47	1.97	1.53	1.42	.84	1.74	1.19
1949	1.55	2.08	1.66	1.57	.86	1.78	1.25
1950	1.45	1.75	1.54	1.34	.80	1.73	1.32
1951	1.53	1.90	1.58	1.36	.79	2.04	1.33
1952	1.63	2.17	1.66	1.45	.79	2.13	1.30
1953	1.59	2.04	1.60	1.47	.81	2.06	1.30
1954	1.62	2.25	1.54	1.55	.83	2.17	1.30
1955	1.50	1.94	1.45	1.35	.82	1.89	1.28
1956	1.56	2.08	1.48	1.41	.87	2.00	1.26
1957	1.60	2.18	1.55	1.57	.87	1.93	1.20
1958	1.63	2.38	1.58	1.64	.83	2.09	1.16
1959	1.45	2.00	1.44	1.40	.77	1.82	1.10

[a] Average inventories divided by average monthly sales *Sources:* Department of Commerce; The Conference Board

Chart 8

Inventory-sales ratios in trade and manufacturing. Annual ratios and period averages by sector and by industry,[a] 1939-1959

(Ratios)

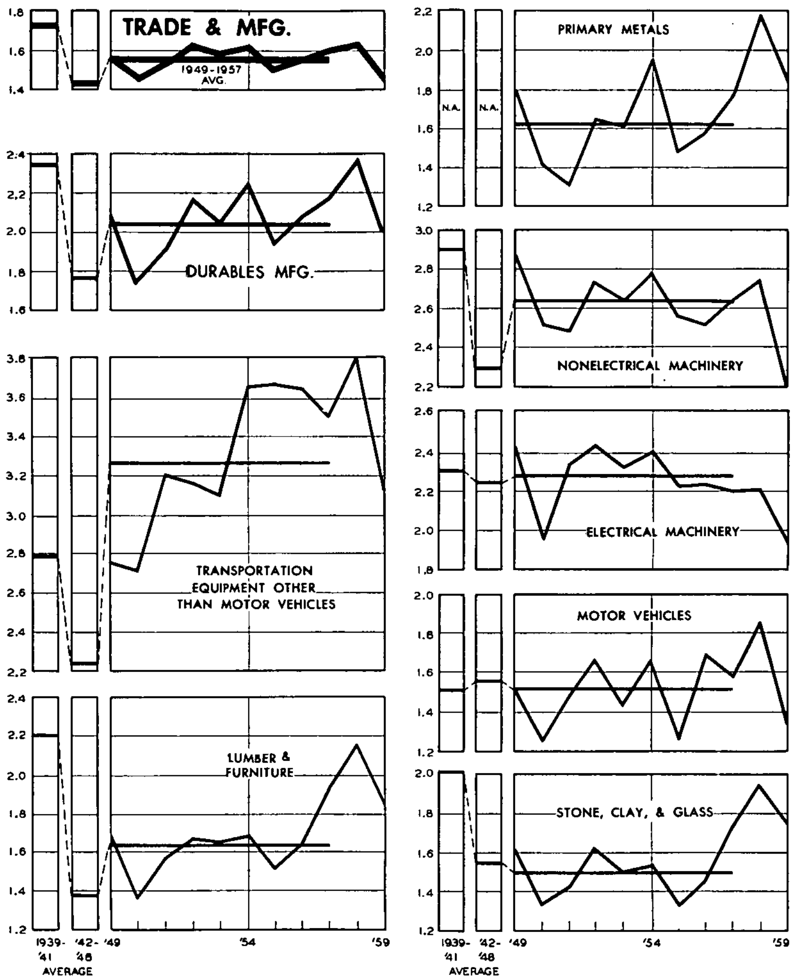
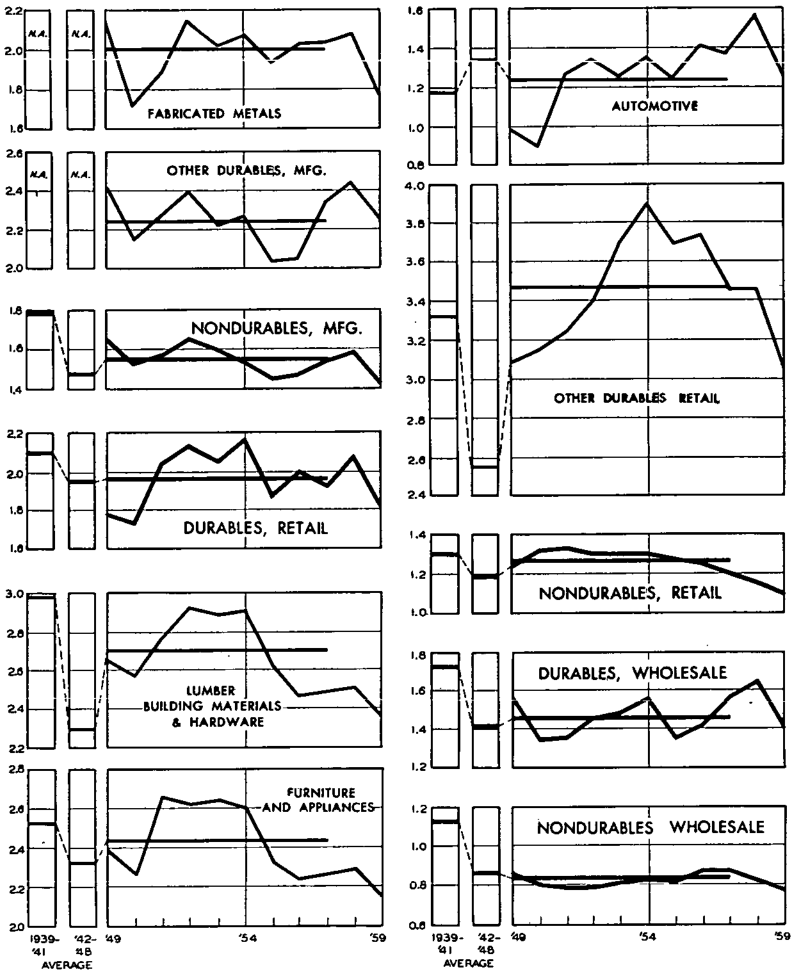


Chart 8 (continued)



^Annual ratios: average of the beginning and the end of the year inventory book values divided by the average monthly sales for the year. Period averages: simple averages of the annual ratios for the period

Sources: Department of Commerce; The Conference Board

exceeded the growth of output, as production and marketing processes have grown longer and more complex. But over the shorter periods that are relevant for cyclical analysis the impact of these divergent growth trends has been negligible. (The inventory-sales and the inventory-output ratios are shown for comparison in Chart 7. For trade and manufacturing, the ratios are given for the years 1939-1958; for manufacturing alone, for the period 1929-1958.)

The ratio of inventories to sales for trade and manufacturing represents an average relationship for the economy as a whole. Behind this average lies a great variety of inventory-sales ratios in different lines of trade and manufacturing, experiencing different, and at times divergent, movements. Table III shows

TABLE IV

TRADE AND MANUFACTURING INVENTORY-SALES RATIOS IN SELECTED ACTIVITIES, 1949-1957 AVERAGE

Activity	Ratio
All other durable goods stores (retail)	3.48
Transportation equipment exl. motor vehicles manufacturing	3.27
Lumber, building, hardware stores (retail)	2.70
Nonelectrical machinery manufacturing	2.64
Furniture and appliance stores (retail)	2.44
Electrical machinery manufacturing	2.29
Other durable goods manufacturing	2.24
Durables manufacturing	2.04
Fabricated metal manufacturing	2.00
Durable goods stores (retail)	1.97
Lumber and furniture manufacturing	1.63
Primary metals manufacturing	1.62
Trade and manufacturing total	1.56
Nondurables manufacturing	1.56
Motor vehicles and parts manufacturing	1.51
Stone, clay, and glass manufacturing	1.50
Durable wholesale trade	1.45
Nondurable goods stores (retail)	1.27
Automotive (retail)	1.24
Nondurable wholesale trade	.83

Sources: Department of Commerce; The Conference Board

the inventory-sales ratios for six major divisions of manufacturing and trade for the years 1939-1959. Even as between the broad classes shown, there have been substantial differences in the levels of the ratios, and while there were similarities in their movements over time, there were also considerable differences.

The greatest fluctuations are found in the ratios for durables manufacturing and for durables retail trade. When these two sectors are split into component industries, further differences in the inventory-sales ratios come to light. These differences are highlighted in Table IV, which shows the average postwar levels of inventory-sales ratios in various types of business activity. The highest ratio ("all other durables goods stores," which includes, among others, camera, jewelry, bicycle stores, etc.) is about four times as high as the lowest ratio in the table (nondurables wholesale trade).[1] A great variety in the movement of inventory-sales ratios also appears. Chart 8 shows the movements of the ratios for the total and six major divisions of trade and manufacturing, and for the component industries of durables manufacturing and durables retailing. Average ratios are shown for three periods: 1939-1941, 1942-1948, and 1949-1957; for the postwar years 1949-1959 the ratios are shown annually.

Generally speaking, inventory-sales ratios were relatively high in the prewar years. They declined substantially in wartime and rose again after the war, but not enough to offset the preceding decline. In most cases, the postwar ratios run considerably below prewar levels.

A comparison among prewar, wartime and postwar average levels is possible for seventeen of the twenty ratios shown in Chart 8. In all but two cases, average levels of inventory-sales ratios declined from 1939-1941 to 1942-1948. (Motor vehicles and parts manufacture and automotive retailing were the two exceptions.) Between 1942-1948 and 1949-1957, thirteen averages rose and only four (the two automotive groups, stone,

[1] As the analysis is extended into greater detail of industry definition, and beyond it into the study of individual companies, an enormous variety of experience appears. A recent survey (*Dun's Review of Modern Industry*, November, 1959, p. 57) finds, for instance, that in 1958 the median inventories-sales ratio in fifteen nondurable wholesale lines included in the survey varied by a factor of 14; from 0.18 (for fresh fruits and produce) to 2.45 (for paints, varnishes and lacquers). Of the fifty-one fresh fruits and produce businesses surveyed, 25% had inventory-sales ratios lower than 0.11, and another 25% had ratios higher than 0.62. (In form, these ratios are comparable to those in Table III, and were derived from published median ratios of annual sales to the value of inventories.)

clay and glass manufacturing and nondurables wholesaling) declined.

In the postwar period, thirteen of the inventory-sales ratios averaged lower than in the prewar years 1939-1941, three were higher and one remained unchanged. The range of differences between prewar and postwar average levels runs from a rise of 17% (manufacturing of transportation equipment other than motor vehicles and parts) to a decline of 26% (nondurables wholesaling).

Within the postwar period itself there have been diverse movements in inventory-sales ratios. While the aggregate ratio for manufacturing and trade changed only slightly over the period, there was a substantial upward trend in durables manufacturing; a marked downward movement for nondurables manufacturing; no visible secular drift in either durables or nondurables wholesaling; a mixed tendency for durables retailing, with a sharp rise in the early Fifties, followed by a decline; and a pronounced downward movement in the nondurables retail sector.

In durables manufacturing, the greatest postwar increase in the inventory-sales relationship occurred in "transportation equipment other than motor vehicles," an industry group dominated by aircraft manufacturing but also including ships, boats and railroad equipment. Primary metals, motor vehicles and parts, lumber and furniture manufacturing, and stone, clay and glass products also exhibited an over-all upward trend. In miscellaneous durables manufacturing and in fabricated metal products, the inventory-sales ratios fluctuated without any pronounced trend; in the two machinery groups, the ratio exhibited a downward tendency. In the durables retail category, the automotive retail ratio showed a consistent postwar rise, while the other three subgroups in this total experienced a rise in the early part of the postwar period followed by a decline.

The Role of Changing Business Mix

All of the diverse movements of the industry ratios are summarized in changes in the over-all ratio. Any change in the inventory-sales ratio for trade and manufacturing may thus be viewed as the outcome of two separate influences: changes in the inventory-sales ratios of the component industries and changes in the relative importance of the component industries.

The magnitudes of the two effects can be estimated separately by (1) computing the over-all inventory-sales ratio in the given year relative to a base-year ratio, on the assumption that only the inventory-sales ratios of the component industries have changed (that is, with the relative importance of these industries held constant); and (2) assuming that only the relative importance of industries changed (that is, holding the ratios in individual industries constant). The calculated product of the two effects approximates the actual relation between the given year and the base-year ratio.

Between 1939 and 1957, the inventory-sales ratio for trade and manufacturing declined from 1.81 to 1.60, or about 12%. This decline may be traced entirely to declines of the individual ratios of the component industries. In fact, were it not that the industries which are relatively "inventory intensive" (industries with relatively high inventory-sales ratios) experienced greater growth than the industries that are relatively light users of inventories, the drop in the over-all ratio for trade and manufacturing would have been considerably larger. Calculations made along these lines and based on the breakdown of trade and manufacturing into fifteen component industries suggest that if all industries grew at the same rate as the total, and their inventory-sales ratios changed as they did, the over-all ratio would have declined by 17% to 18%, rather than by 12%. On the other hand, if the ratios of the component industries were the same in 1957 as they were in 1939, the over-all inventory-sales ratio would be 5% higher in 1957 than it actually was.

The same analysis applied to the changes from 1948 to 1957 yields somewhat different results. Here, both effects turn out positive. The aggregate inventory-sales ratio rose 8.8%; of this rise, changing component ratios contributed 5.4%, and the mix effect (change in the relative importance of industries) 3.2%.

IV. A Profile of Inventory Fluctuations

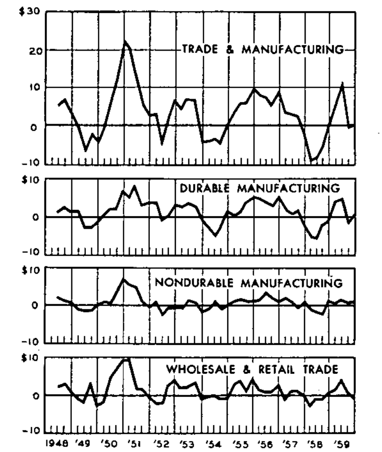
The rates of inventory change in all three major subdivisions of business – durables manufacturing, nondurables manufacturing, and trade – show definite cyclical swings, but the magnitude of these swings has differed considerably. As may be seen from Chart 9, the greatest fluctuations have consistently occurred in the durables manufacturing sector. Durables manufacturing may, in fact, be viewed as a principal locus of the variation in business inventories. Compared with durables manufacturing, the cycles in the rate of change of trade inventories have been notably smaller, and those in nondurables manufacturing smaller still.

No outstanding differences appear to exist in the timing of quarterly cyclical turning points for the above three groups of inventories. However, a comparison of fluctuations in manufacturers' inventories

Chart 9

Quarterly changes in trade and manufacturing inventories, by sector, 1948-1959▲

(Billions of dollars)



▲Seasonally adjusted annual rates
Sources: Department of Commerce; The Conference Board

at the various stages of fabrication brings to light certain differences in the timing of fluctuations. Chart 10 shows the quarterly changes in manufacturing inventories at the three stages of fabrication – purchased materials, goods in process, and finished goods – together with the changes in trade inventories. Except for the period 1950-1951, when the quarterly changes in the goods-in-process inventories were smaller than for the other kinds of stock, the magnitudes of movements of all four series have been roughly similar. The outstanding variation in timing is that the peak rates of accumulation and liquidation of manufacturers' finished-goods inventories occurred considerably later than in the other three types of inventories.

Diffusion of Inventory Change

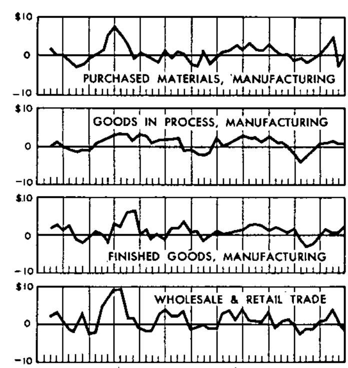
Because of the manifold variety of specific operating conditions prevailing in each industry, one would hardly expect that inventory series for a large number of industries should move in uniform fashion. But the record shows that in times when business inventories as a whole have moved rapidly, inventories in a large majority of industries have moved in the same direction as the total.

Chart 11 indicates that the correlation between the proportion of total inventory series rising and the rate of change in total inventories has been remarkably high. The percentage of series rising shown in the chart represents the number of industries in which inventories rose during a given month, divided by twenty-seven, the total number of component industries. In effect, the measure is a diffusion index of inventory behavior. The result suggests that the phenomenon of inventory change is broadly and rapidly diffused throughout the general business scene; it is rare that any small group of industries dominates total inventory movement. When total inventories have been declining, inventories in most industries have also declined; when total stocks have been increasing, inventories in most industries have also increased.

Chart 10

Quarterly changes in trade and manufacturing inventories, by stage of fabrication, 1948-1959▲

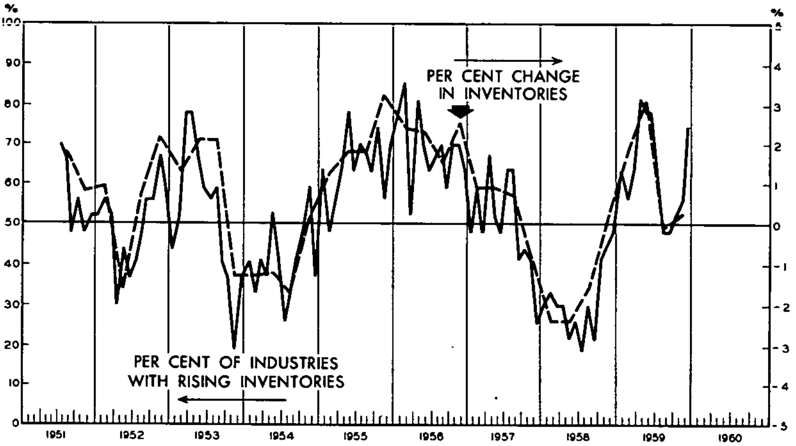
(Billions of dollars)



▲Seasonally adjusted annual rates
Sources: Department of Commerce; The Conference Board

Chart 11

Relationship between the rate of inventory change and the diffusion of inventory increases by industry,[▲] 1951-1959



▲Based on seasonally adjusted data. The diffusion measure is the number of inventory series which have increased during the month as percentage of the total number (27) of trade and manufacturing series. It is shown quarterly

Sources: Department of Commerce; The Conference Board

In the majority of *individual* industries, the cyclical movements of inventory stocks have paralleled the movements in total business inventories. This is illustrated in Chart 12, in which the level of inventories in twenty-seven trade and manufacturing industries is shown for the dates that marked the peaks, the troughs, and points one year after the trough, for each of the postwar cycles in total trade and manufacturing inventories.

Movements in Composition of Inventories

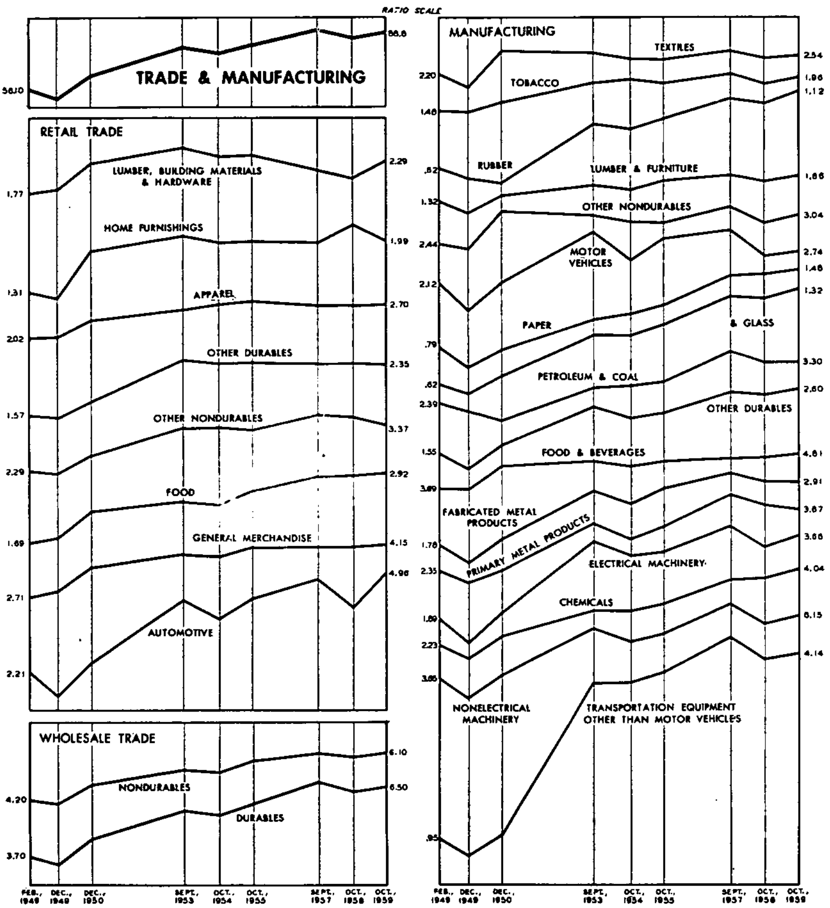
The over-all similarity of inventory cycles in different areas of the economy furnishes one indication that normally no great violence is done to the facts when business inventories are treated as a whole rather than as a collection of parts. Nevertheless, there appear to be some systematic differences in amplitude and timing among specific inventory cycles. These differences produce a characteristic set of changes in the composition of inventories at different stages of the cycle in total inventories. The share in total inventories typically held by industries that are highly sensitive to the business cycle tends to decline substantially when total inventories contract, and then tends to recover, either in the earlier or later stage of the expansion in aggregate inventories. Automobile retailers, motor vehicle manufacturers, and machinery and metal producers are in the class of cycle-sensitive industries. At the other end of the scale are industries that normally expand their share of the total during the contraction phase of total inventories, because they are little affected by the business cycle. Among these are food manufacturers, food stores, apparel retailers and general merchandise stores. Most industries are somewhere in the middle. The typical pattern for each of the twenty-seven industry groups is mapped in Chart 13, which shows the average share of each industry in total business inventories at different stages of the postwar inventory cycles.

The characteristic patterns of distribution of business inventories by stage of fabrication are shown in Chart 14. Manufacturers' stocks of materials and

Chart 12

Inventories in trade and manufacturing industries at the various stages of postwar cycles in total business inventories ▲

(Billions of dollars)



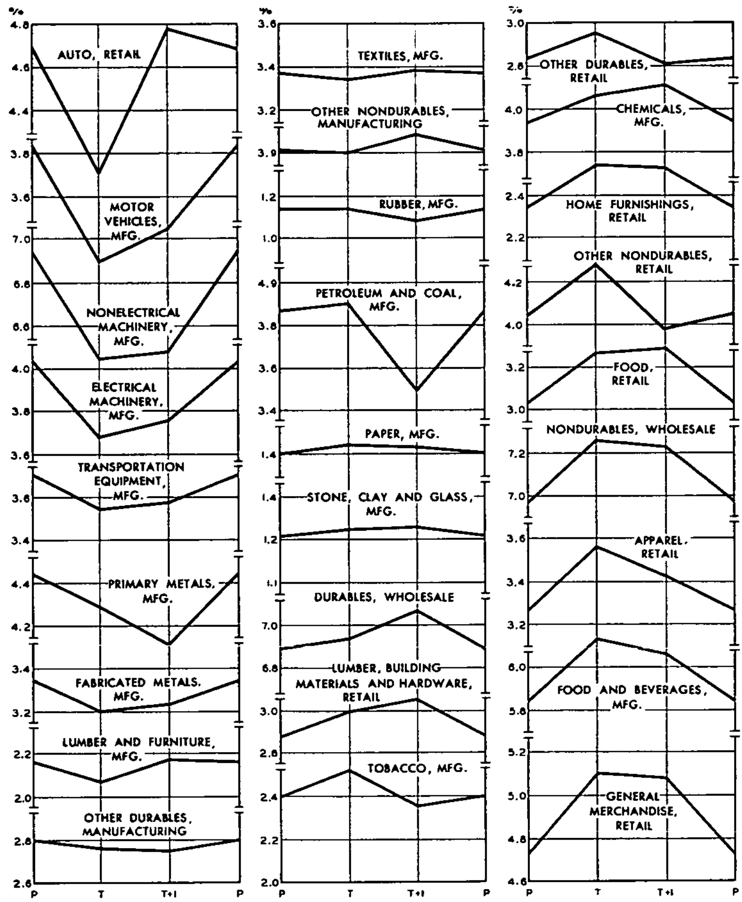
▲Seasonally adjusted book values. The dates shown represent peaks (Feb. 1949, Sept. 1953 and Sept. 1957), troughs (Dec. 1949, Oct. 1954 and Oct. 1958) and the month one year after the trough in the postwar cycles of trade and manufacturing inventories

Source: Department of Commerce

Chart 13

Average percentage distribution of business inventories by industry at the different stages of the postwar cycles in total business inventories ▲

(Per cent of total inventories)



▲Based on seasonally adjusted data. The average percentage distribution is shown for the peaks (Feb. 1949, Sept. 1953 and Sept. 1957), troughs (Dec. 1949, Oct. 1954 and Oct. 1958) and for one year after the troughs

Sources: Department of Commerce; The Conference Board

goods-in-process inventories typically experience a drop in shares during the contraction phases (of inventories) and a compensating gain during expansions. However, the gain in the share of materials was concentrated in the early stages of the expansions, while the bulk of increase in the share of goods-in-process stocks occurred late in the expansions.

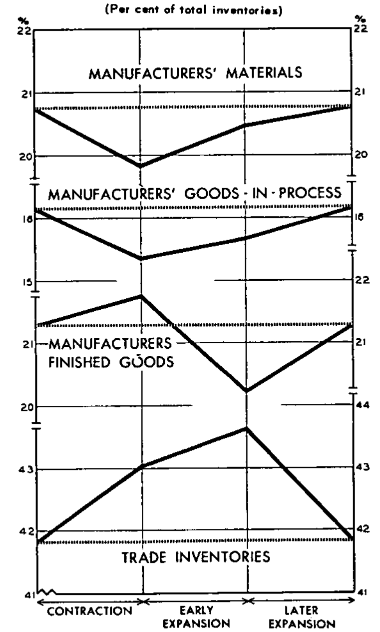
By contrast, the proportion of total inventories made up of manufacturers' finished goods tends to rise during the contraction of total inventories, reflecting both an accumulation of finished goods in the face of reduced sales and a relatively sharp drop in goods-in-process stocks occasioned by curtailment of production rates. This share tends to fall sharply in the early recovery as production and sales revive. Finally, it rises again, over the later parts of the expansion.

The share of retail and wholesale trade inventories, on the other hand, advances throughout the contraction phase and the early stage of expansion, and then declines sharply in the later phase of expansion.

Chart 14

Average percentage distribution of business inventories by stage of fabrication at different stages of the postwar cycles in total inventories[▲]

(Per cent of total inventories)



▲Based on seasonally adjusted data. The average percentage distribution is shown for the peaks (Feb. 1949, Sept. 1953 and Sept. 1957), troughs (Dec. 1949, Oct. 1954 and Oct. 1958) and for one year after the troughs

Sources: Department of Commerce; The Conference Board

V. Factors Influencing Inventory Change

The measurement of the relative adequacy of existing inventories is, in a sense, a measurement of the probable future course of inventory demand; the key to prediction of inventory movements of the future thus lies in quantitative explanation of past levels of inventories, and changes in those levels.

The prospective behavior of inventories is developed here from study of the postwar behavior of aggregate inventories; the fact that inventories in different industries have moved largely together over the postwar cycles indicates that the aggregate treatment is suitable to the present purpose. However, changes in the composition of inventories cannot be entirely excluded from the analysis, for such changes often have significant effect on the short-term movements in inventories. This issue of "mix," therefore, is taken into account explicitly, although certainly not exhaustively, in the analysis that follows.

Influences that govern the movements of inventories are manifold, and no single index can adequately represent them. All the following questions have a definite bearing on the course of inventories and might well be taken into account in any appraisal of the inventory situation.

1. How high are inventories, absolutely and relative to sales?
2. What has been the recent performance of new orders? Have they been running ahead of or behind sales?
3. How high are unfilled orders, absolutely and relative to sales?
4. Where are prices heading? In what state are price anticipations, and what will be their effect on *real* inventory holdings?
5. What is the secular rate of growth of inventories?
6. In what direction is the composition of industries moving? Are inventory-intensive industries

(those with high inventory-sales ratios) increasing in importance relative to industries with low inventory-sales ratios?

7. How will the financial situation affect inventories? What is the liquidity position of business? How high are interest rates?

This list suggests a selection of variables to be used in a statistical analysis. The influence of each of several variables drawn from this list of questions on inventory demand is discussed below, and their simple relationships with inventory change are examined. Later, an over-all evaluation of influence of each variable on inventory formation is presented.

The Inventory-Sales Relationship

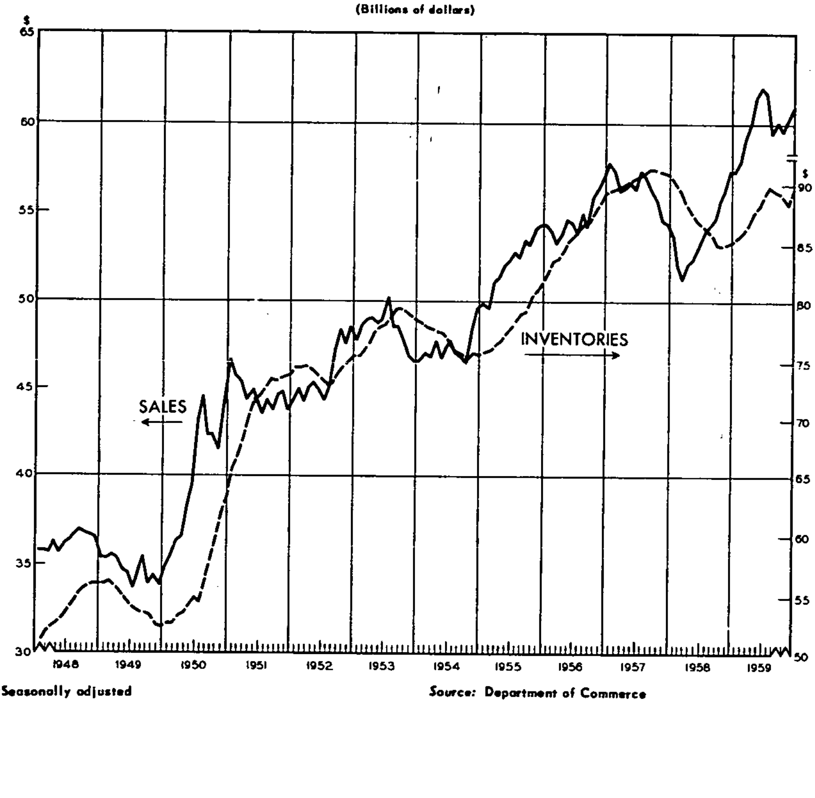
Businesses which hold inventories of goods intend to sell them, either in the same form or transformed by manufacturing activity, within reasonable time. A substantial correspondence between sales and inventories is, therefore, to be expected. As Chart 15 shows, the movements of business inventories follow those of sales, though, as one might expect, inventories lag behind sales by several months. This relation is pronounced.

A technique that The Conference Board has been using with good results for durables manufacturing inventories involves a projection of book values of

Chart 15

Trade and manufacturing sales and inventories, monthly, 1948-1959 ▲

(Billions of dollars)



▲Seasonally adjusted

Source: Department of Commerce

manufacturing inventories at each of the three stages of processing separately from the past pattern of sales alone, and then a summation of the three forecasts to obtain calculated total inventories for durables manufacturing.[1]

Another starting point is the inventory-sales ratio. For practical purposes, this ratio may be viewed as a critical control mechanism in the goods-producing and distributing sector. The ratio may be considered as indicating the gap between the actual and an "ideal" inventory situation. The size and the direction of the gap may then be related to the intensity of the effort exerted to reduce it. When the over-all inventory-sales ratio is relatively high, it seems reasonable to suppose that the pressures to reduce the rate of inventory accumulation and perhaps then to cut stocks are much stronger and more widely diffused among business firms than in a situation when the ratio is at or near a "norm." Conversely, a relatively low ratio suggests, in general, incentives to increase inventories. The more the ratio departs from "normal," the stronger, presumably, are the incentives to accumulate.

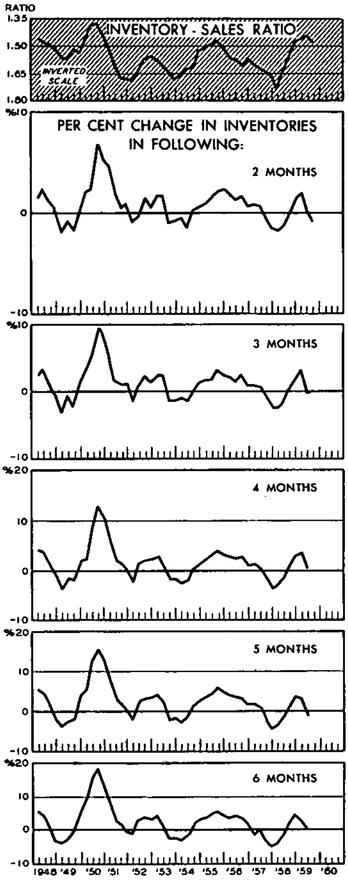
It should be obvious that inventory decisions are first of all governed by ratios of individual firms. But the inventory cycle is a pervasive phenomenon, and the cyclical movements of inventory-sales ratios at the industry level generally tend to conform to the movements of the over-all ratio. One may expect to find a further correspondence with the over-all ratio at the company level. It is reasonable to suppose that a preponderant majority of company ratios are high when the aggregate ratio is high, and low when the aggregate ratio is low. As a result, the aggregate ratio should ordinarily provide a reliable guide to the kinds of inventory decisions being considered in the average firm.

It is not surprising, therefore, that an inverse correlation exists between the level of the inventory-sales ratio as of a given point in time, and the rate of inventory accumulation in an ensuing short period. Such a correlation, as Chart 16 indicates, is found for ensuing periods of two, three, four, five and six months. The high degree of correlation of subsequent changes in inventories with the existing inventory-

[1]The lag of inventories behind sales was found to be different for inventories at each of the three stages of processing. The lags actually used were determined by choosing in each case the lag periods that yielded highest correlation coefficients. For purchased materials stocks, total sales three, four and five months earlier were taken, for goods-in-process inventories sales four, five and six months earlier, and for finished goods the lag was seven, eight and nine months. For further detail see *The Business Record*, May, 1958, pp. 200 ff.

Chart 16

Inventory-sales ratio and the per cent change in business inventories in an ensuing short period.▲ Trade and manufacturing, 1948-1959



▲Based on seasonally adjusted data. The inventory-sales ratio is shown as of the beginning of each calendar quarter

Sources: Department of Commerce; The Conference Board

sales ratio suggests that it is feasible to use the inventory-sales ratio for statistical prediction of changes in inventories. This correlation reflects a recognizable economic relationship, and not a mere statistical association. The ratio of inventories to sales should thus be one of the main building blocks of a short-term expression of inventory demand.

New Orders

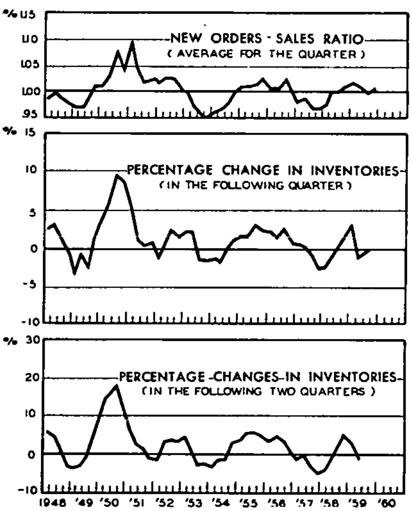
The rate of new orders is a second obvious influence on attitudes toward inventory. The rate is expressed here relative to sales (that is, as a new orders-sales ratio) in order to remove the trend effects present in the absolute amounts.[1]

The rate of new orders relative to the rate of sales serves as a general foreshadowing indicator of business conditions. By the same token, it indicates future demand for inventories. This is so because the desired level of inventories is intimately related to the prospective level of activity. One would expect that when new orders are running above sales, and the reservoir of future business is built up, an accumulation of inventories becomes desirable, as the planned production rate rises to fill these orders. The subsequent increase in the actual production rate entails a rise in inventories concentrated in the in-process stocks.

A definite correlation should thus exist between the ratio of new orders to sales and the rate of inventory formation in an ensuing period. Chart 17 shows that there has been a close correspondence between the average ratio of new orders to sales in a given quarter and the rate of inventory change in both the following quarter and the following six-month period. The new-orders ratio thus modifies the influence of sales on inventory policy; at any given level of sales, inventory demand will be greater if new orders are high, and less if new orders are low.

Chart 17

New orders-sales ratio and the percentage change in business inventories in the ensuing three and six months periods.▲ Trade and manufacturing, 1948-1959



▲Based on seasonally adjusted data
Sources: Department of Commerce; The Conference Board

Price Movements

Change in prices enters into the analysis of inventory conditions in two distinct ways. In the first place, revaluation of existing inventories because of price changes affects the statistics on which the analyst has to rely, and the resulting disturbance needs to be measured and adjusted for. This is the purpose of the inventory valuation adjustment in the national accounts. Secondly, changes in prices, and in particular the expectations of future price changes that often accompany a present change in prices, create incentives for inventory accumulation and liquidation. These are, of course, the incentives to maximize inventory profits and to minimize inventory losses by adjusting purchasing to anticipated movements of prices.

Chart 18 shows that there is an appreciable degree of positive correlation between movements of prices and concurrent changes in inventory values. The reasons for the correlation between changes in prices

[1]New-orders information is published for manufacturing only. For the sake of consistency, a ratio of new orders to sales for all trade and manufacturing was estimated for use throughout this study. The estimate was made simply by assuming that, in the trade sectors, sales equal new orders over a three-month period. (This assumption is clearly justified for retail trade, and is probably not far from the mark for the trade sector as a whole.) The aggregate ratio used here is thus manufacturing new orders plus trade sales divided by manufacturing and trade sales

and in inventories, however, are not entirely clear; an exhaustive treatment would require an analysis in terms of "real" (*i.e.*, constant-dollar) inventories and inventory changes.[1] The simplified treatment given here yields a few matters of interest, however. It is quite clear that rapid changes in prices have been accompanied by parallel movements in inventory values. The distortion introduced into the inventory data by the revaluation of inventory within an accounting period may be removed by adjusting for inventory valuation. The magnitude of the inventory valuation adjustment, expressed relative to the volume of inventory stock, indicates the approximate degree to which the price change has simply induced a change by revaluation.

When these valuation consequences of price changes are removed, a certain parallel to price changes still exists in the inventory data. Moreover, the chart indicates that there has been some lead of price changes over the rate of inventory accumulation, regardless of whether the valuation adjustment has been removed. This lead might be considered as evidence of the fact that a "speculative" element exerts an independent influence on inventory plans.

The importance of this influence is difficult to measure, however. In the postwar years, the rate of price change has been highly correlated (inversely) with inventory-sales ratios, and (positively) with new orders-sales ratios, and its independent role might be masked by these other variables. It may be that price expectations latently carry a much greater significance for inventory policy than can now be established from the statistical record.

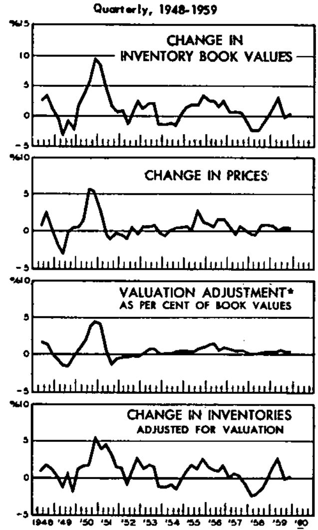
Changes in Composition of Business

In most of this paper, differences between industries are ignored in favor of treatment of aggregates. There are, however, significant differences among industries that have a bearing on the behavior of aggregate inventories. Especially relevant for the present purpose are interindustry differences in inventory-sales ratios. Any given volume of sales may require very different levels of inventories to support it, depending on how inventory intensive are the goods being manufactured and sold.[2] Similarly, a given percentage change in the volume of sales may generate widely different percentage changes in aggregate inventories, depending on the industry in which the change in sales is concentrated. While the industrial composition of activity does not fluctuate violently in the short run, changes in the industry mix such as have occurred typically in the course of a few months have been significant enough to merit consideration.

Chart 18

Changes in inventory book values and in the industrial wholesale price index, and the relative size of inventory valuation adjustment ▲

Quarterly, 1948-1959



*For convenience of exposition, the sign of inventory valuation adjustment here is opposite to that conventionally given by the Department of Commerce. The quarterly IVA for trade and manufacturing was estimated from data published by the Department of Commerce

▲ Based on seasonally adjusted data

Sources: Department of Commerce; Department of Labor; The Conference Board

[1] An article on physical levels of business inventories will appear in a forthcoming issue of *The Conference Board Business Record*

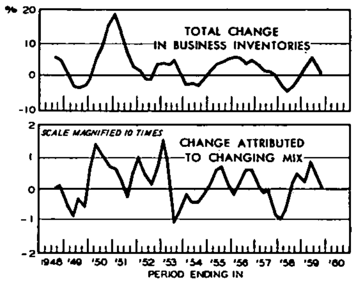
[2] Given the actual volume of business sales ($56 billion per month) in 1957, "normal" inventories would have been $67 billion if activity consisted entirely of selling food, clothing, gasoline, cosmetics, etc. (nondurable retail trade) and $198 billion if all activity were directed to manufacturing boats, planes and bicycles (transportation equipment other than motor vehicles). Actual business inventories averaged $90 billion in 1957.

Estimates suggest that, on the average, about 15% to 20% of the six-month changes in inventories in the postwar period may be attributed to changing mix.[1]

Chart 19

Total change in business inventories and change attributed to changing mix. Six months periods by quarter, 1948-1959▲

(Per Cent)



▲*Based on seasonally adjusted data*

Sources: *Department of Commerce; The Conference Board*

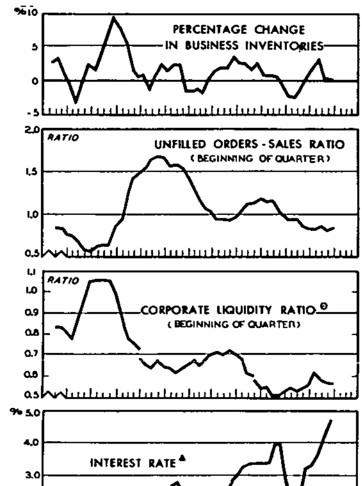
Since in about three cases out of four, the change indicated by changing mix was in the same direction as the total change, one may conclude that changes in mix have accentuated rather than mitigated inventory fluctuations. This apparently results from the fact that inventory-intensive industries have experienced wider cyclical swings than industries with lower ratios of inventories to sales. An estimate of inventory changes attributed to changes in industry mix is shown together with total inventory changes in Chart 19.

Unfilled Orders

Earlier in this section, the influence of the new orders-sales ratio on the rate of inventory demand was discussed, and indications of a relationship were obtained. The ratio of new orders to sales was interpreted as an indicator of *change* in the reservoir of

Chart 20

The rate of inventory accumulation compared to the unfilled orders ratio, corporate liquidity ratio, and the interest rate. Quarterly, 1948-1959



©Ratio of cash and U.S. Government securities to current liabilities, mfg. corporations

▲Prime commercial paper, 4-6 months

Sources: Department of Commerce; Federal Reserve Board; Federal Trade Commission; Securities and Exchange Commission; The Conference Board

---

[1] This estimate was arrived at on the basis of a breakdown of trade and manufacturing totals into seventeen industries, by permitting the relative distribution of total sales by industry to vary while total sales and the industry inventory-sales ratios were held constant at their actual levels in 1957, the base year. In thirty-three out of the forty-five six-month periods studied, the indicated inventory change owing to changing mix was in the same direction as the total change. Among these thirty-three cases, the changes due to changing mix averaged 15% of total changes.

The correlation coefficient (r) between the total changes in inventories and the changes attributed to changing mix is .44. This provides a means for an alternative estimate of the contribution of mix effect to total inventory changes; the explained portion (r^2) of inventory change is 19% ($.44^2$), which checks reasonably with the 15% arrived at above future work. The absolute size of this reservoir itself, as it may affect the rate of inventory accumulation, is reflected in the ratio of unfilled orders to sales. No simple relationship is apparent between the ratio of unfilled orders to sales[1] and the total changes in inventories, as shown in Chart 20. (However, unfilled orders are closely correlated with the variation in inventories not explained by sales and orders: see below.

Liquidity and Interest Rates

In addition to the factors mentioned above, influences on the behavior of inventories may be sought in financial statistics.

At least two dimensions of the financial environment might well receive attention: the availability of funds to finance inventories and their cost. Availability of funds for additional inventory investment may be gauged indirectly from the ratio of cash assets to current liabilities of businesses, and the cost of money is of course reflected in the interest rate. In Chart 20 the liquidity ratio and the interest rate are plotted against changes in inventories.[2]

There has been some similarity between the major cyclical movements in corporate liquidity and in inventory accumulation, but the long-term trends have been different, and it is not clear that the parallel reflects a consistent causal relation.

Similarly, there is little indication of (inverse) correlation between inventory movements and the interest rate. This obviously does not exclude interest rates as a variable influencing inventory policy, and perhaps a quite important variable. Its historical role in the postwar years, however, is not readily apparent.

VI. Prediction of Inventory Change

Each of the several economic series reviewed above illuminates a special aspect of the inventory picture. In broad outline, business inventories follow the movements of sales, with a lag of several months, but considerable departures from that general relation have occurred at times. The level of business inventories is apparently influenced not only by the absolute level of sales, but also by the composition of the sales total by industry. Thus changes in the business mix apparently account for roughly 15% of short-term changes in business inventories. It is possible that the importance of changing mix would appear to be still greater if a more detailed industry breakdown were available.

The explanation of inventory conditions through sales provides useful first approximations of inventory demand. A more complete explanation of inventory fluctuation may be obtained with a set of more specific indicators. In this paper, a number of such indicators are related to inventory changes. These are the inventory-sales ratio, the ratio of new orders to sales, the change in a price index of industrial commodities, the ratio of unfilled orders to sales, an index of industry "mix," an index of corporate liquidity, and the interest rate.

Both the inventory-sales ratio and the ratio of new orders to sales are clearly related to inventory fluctuations. What is more, these turn out to be lag relationships, of such a nature that it seems possible to predict, with some accuracy, inventory movements as much as three or six months in advance, given the current ratios of inventories to sales and of new orders to sales. Price changes also are related to inventory changes. Apparently this relationship is mainly built upon the induced effect of price changes on inventory valuations – the "inventory valuation adjustment" – but a mild indication of a lead of price movements over inventory changes also appears. As has been mentioned above, change in the business "mix" may have some influence on the rate of inventory accumulation. There is some suggestion of a relationship of the corporate liquidity ratio to inventory movements, but no indication of a direct relationship was found for either the interest rate or the ratio of unfilled orders to sales.

These results were obtained by relating the variables one at a time to inventory changes. Multiple regression analysis provides a method for integrating the findings and for estimating the relative importance of each indicator for inventory movements. By this means, the net effect of interrelationships among the different indicators may be found, redundant relationships cleared away, and relations that otherwise might be obscured brought to light. The product of multiple regression analysis is an estimating equation that gives a predicted or "normalized" value of the

[1] As in the case of the new orders-sales relationship, the ratio of unfilled orders to sales used here represents total trade and manufacturing activity. It is assumed that the backlog of unfilled orders in the trade sector is zero; manufacturers' unfilled orders are divided by trade and manufacturing sales combined

[2] Liquidity is represented by the ratio of cash and U. S. Government securities to total current liabilities for manufacturing corporations; the interest rate is the rate on four-to-six month prime commercial paper

dependent variable – in the present case, inventory change – in terms of the specific values of independent variables.

Such an estimating equation is constructed here to explain percentage changes in business inventory book values over successive six-month periods, staggered at quarterly intervals. The three factors that were clearly related to inventory changes – the inventory-sales ratio at the beginning of the period, the average ratio of new orders to sales during the quarter immediately preceding the period, and the percentage change in the index of industrial wholesale prices in each of the six-months periods under analysis – were, of course, included in the scheme. The interest rate (prime commercial paper, 4-6 month) and the percentage change in inventories indicated by an index of industry mix, calculated as explained earlier in this section, were also included in initial formulations of the equation. Other indicators – corporate liquidity, the ratio of unfilled orders to sales, and the rate of price change in a preceding period – were left out of the initial formulation, and held for comparison with the "residuals" of the equation, that is, with the differences between the rate of inventory change indicated by the equation and the actual change. Such comparisons provide an indication of whether a variable which was excluded from the equation is likely to make a net contribution to the explanation if it is added to the scheme.

The estimating equation which was obtained for six-month business inventory changes appears as follows:

$$Y = -121.13 - \underset{(3.84)}{18.48X_1} + \underset{(9.73)}{72.51X_2} + \underset{(.111)}{.788X_3} - \underset{(.461)}{.028X_4} - \underset{(.310)}{.026X_5}$$

where:

Y is the percentage change in inventory book values for trade and manufacturing over a six-month period,

X_1 = Inventory-sales ratio at the beginning of the period

X_2 = Average ratio of new orders to sales for the quarter preceding the period

X_3 = Percentage change in the index of industrial wholesale prices, during the period

X_4 = Percentage change in inventories attributable to changing mix, during the period

X_5 = Interest rate on four-to-six months' prime commercial paper: average for the first quarter of the period

The numbers written in parentheses under the regression coefficients are the standard errors of these coefficients. The ratio of a regression coefficient to its own standard error provides a test of significance of the coefficient. This amounts to testing whether there is a real correlation between the variable to which the coefficient applies and the dependent variable to be explained. When a probability standard of one chance in twenty is applied, the ratio should equal at least 2.0 if the coefficient is to be accepted as significant. With a much stricter standard of one chance in 100, the ratio must be 2.6 or more.

The coefficients for the first three variables – inventory-sales ratio, new orders-sales ratio and price change – thus appear to be highly significant; those for the other two variables – change indicated by shifting mix and the interest rate – are not significant.

The statistical fit of the equation is excellent. The independent variables explain 91% (R^2 = .91) of the total variation in the six-month changes in inventories that have occurred during the period. Of course, the two variables that turned out to be not significantly related to movements in inventories did not contribute to this explanation. The correspondence between the actual inventory changes and the changes calculated from the equation has been very close.[1]

The residuals yielded by this regression were then compared with the variables that were left out of the equation. These were corporate liquidity, the ratio of unfilled orders to sales, and the rate of price change during the quarter *preceding* the six-month period under analysis. No indications of a possibly important relationship between either the liquidity ratio or the price change during the prior period were obtained. It was found, however, that the ratio of unfilled orders to sales appears to be rather closely correlated with the discrepancies between the actual and the calculated values, suggesting that an additional contribution to the explanation of inventory changes is likely to be obtained by including the ratio of unfilled orders to sales in the analysis. The equation described above was therefore recast, dropping the two insignificant

[1]The greatest discrepancies between actual and calculated values occurred during the 1959 steel strike, which artificially constricted the rate of inventory accumulation

variables, eliminating the price variable, which represents an *ex-post* element,[1] and adding the unfilled orders ratio.

The new equation thus makes use of current information about the ratio of inventories to sales, that reflects the relative deficiencies or surpluses of inventory stocks; the relative size of the reservoir of work that already has been ordered, as reflected in the ratio of unfilled orders to sales; and the rate at which this reservoir is being filled or emptied by the relation of new orders to sales.

Two equations were constructed for prediction of inventories on the basis of lagged data for the three explanatory variables. The two magnitudes to be explained are the percentage changes in trade and manufacturing inventories (adjusted for valuation) over three-month and six-month periods, respectively. The two estimating equations are:

$$Y_1 = -14.59 - 11.26\,X_1 + 30.75\,X_2 + 1.88X_3$$
$$(2.33) \qquad (5.52) \qquad (.57)$$

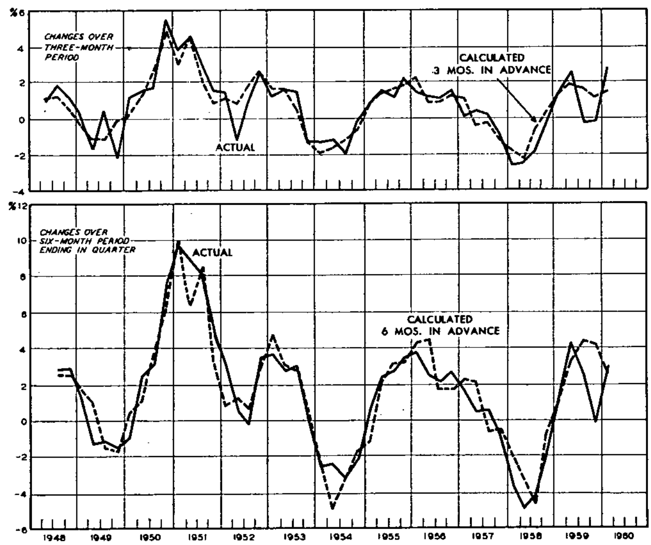
$$Y_2 = -17.64 - 25.26\,X_1 + 55.31\,X_2 + 2.88X_3$$
$$(3.07) \qquad (7.18) \qquad (.74)$$

[1] In order to compensate for the elimination of the price variable from one side of the relation, the inventory valuation adjustment was excluded from the measure of inventory change on the other side. After the adjustment for valuation, the analysis is in terms of changes in inventories valued at current prices. The quarterly inventory valuation adjustment for trade and manufacturing was estimated from the corporate and nonfarm totals published by the Department of Commerce

Chart 21

Percentage changes in business inventories. Comparison of actual changes with changes calculated three and six months in advance,[a] 1948-1960

(Per cent)



[a] Seasonally adjusted data. Changes during the period (after valuation adjustment) as per cent of book values at the beginning of the period

Sources: Department of Commerce; The Conference Board

The Y_1 and Y_2 are the percentage changes in inventories, excluding the valuation adjustment, over three-month and six-month periods; X_1 is the inventory-sales ratio at the beginning of the period, X_2 the average ratio of new orders to sales in the quarter immediately preceding the period, and X_3 the ratio of unfilled orders to sales at the beginning of the period.

The comparison of regression coefficients with their own standard errors reveals that in both equations all the regression coefficients are highly significant. The three-months' equation explains 78% of the variation that has occurred in the changes of business inventories over the corresponding periods, and the six-months formula, 89%. While this explained portion is not so great as the 91% found in the tentative equation, it should be noted that the present formulation makes use only of leading variables; it is accordingly predictive as well as explanatory. The (unbiased) standard errors of estimate were .87 and 1.12, respectively. This means that in two cases out of three the difference between the actual and the estimated percentage change in inventories over three-month periods (with the underlying relationships such as prevailed in the period 1948-1959) may be expected not to exceed .87 percentage points, and over six-month periods will not be more than 1.1 percentage points. The fit of both equations, as shown in Chart 21 is very good.

For interpretation of results it may be convenient to rearrange the formulas somewhat, by changing the units of the variables and measuring them from their period averages rather than from zero as was done for the purpose of computation.[1] As they appear above, the equations, while sound statistically, are slightly awkward. Rewritten, the equations appear as follows:

$$Y_1 = .58 - 11.26 (X_1 - 1.56) + 30.75 (X_2 - 1.00) + 1.88 (X_3 - 1.06)$$

$$Y_2 = 1.31 - 25.26 (X_1 - 1.56) + 55.31 (X_2 - 1.00) + 2.88 (X_3 - 1.06)$$

The first constant term of each equation represents the secular rate of growth in inventories; *i. e.*, the growth that may be expected when all the independent variables are at their respective average ("normal") levels. Because the changes in business inventories are adjusted for valuation and are expressed in percentage terms, it is the secular rate of growth of *real* inventories that is reflected in the regression constants. Both equations yield about the same estimate of the secular rate, about 2.5% per annum.

To each one-point difference in the inventory-sales ratio, there corresponded a .11 percentage point difference of opposite sign, in the rate of inventory accumulation in the ensuing three months and a corresponding .25 percentage point difference for the six-month change. For instance, a five-point increase in the inventory-sales ratio from, say, 1.46 to 1.51 entails a *decrease* in the expected rate of change in business inventories of .6 percentage points over the next three months and of 1.2 percentage points over the next six months.

For each percentage point of excess (or deficiency) of orders over sales (for trade and manufacturing combined) during a given quarter, there has been on the average a .3% additional accumulation (or liquidation) of inventories during the following quarter (an additional .6% accumulation over the ensuing six-month period).

To each 10% difference in the level of unfilled orders relative to sales there correspond a difference in the same direction in the rate of inventory accumulation – a difference of .2% over three-month periods and a difference of .3% for six-month periods.

Estimates of the prospective level of business inventories in dollar terms may be derived from the formulas. As Chart 22 shows, the correspondence between actual level of inventories and the level predicted by the formulas three and six months in advance has been good. (The data underlying the chart are given in Table V.) The projected percentage change is simply linked to the actual book-value amount at the beginning of the period, yielding a value estimate for the end of the period. This estimate, however, does not include the inventory valuation adjustment for the forecast period. For practical purposes, in those periods when prices are stable, the Inventory Valuation Adjustment is likely to turn out small and may be ignored. Alternatively, when price changes are substantial and a forecast of book values at the end of the period is desired, an approximate estimate of the valuation adjustment may be constructed on the basis of actual and expected price trends.[2]

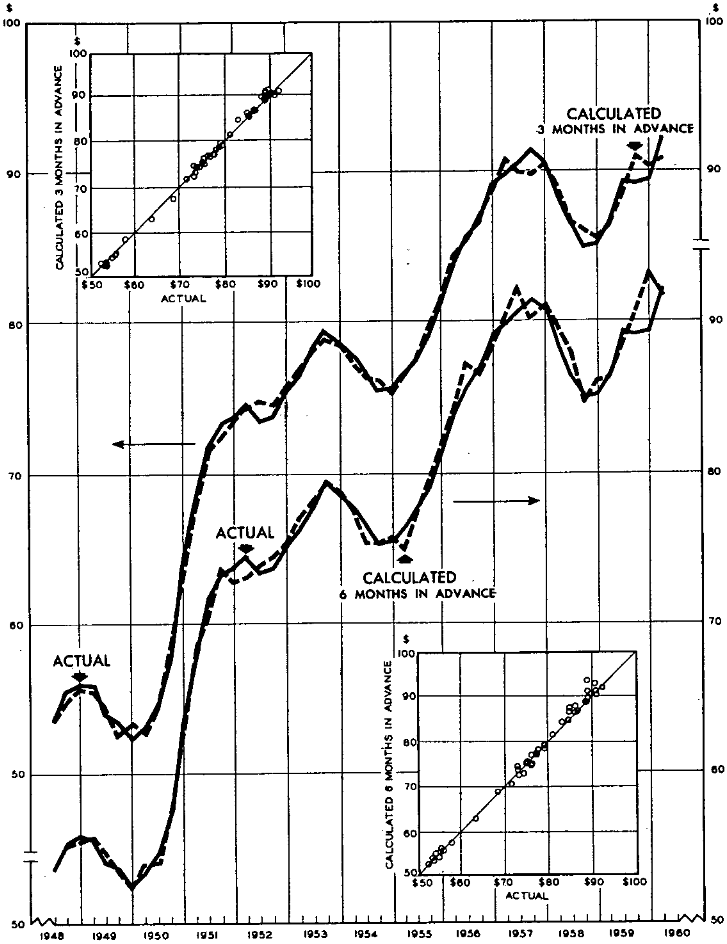
[1] The postwar average for the inventory-sales ratio is 1.56, for the new orders-sales ratio 1.00, and for the ratio of unfilled orders to sales 1.06

[2] For the sake of comparability with historical book-value data, the "calculated" inventory levels shown in Chart 22 and Table V include the actual inventory valuation adjustment. For the most recent dates, valuation adjustment has been estimated from less complete data than for the prior periods

Chart 22

The level of trade and manufacturing inventories: actual, calculated three months in advance and calculated six months in advance.[▲] Quarterly, 1948-1960

(Billions of dollars)



▲Seasonally adjusted book values; for details see Table V

Sources: Department of Commerce; The Conference Board

TABLE V
ACTUAL BUSINESS INVENTORIES AND INVENTORIES CALCULATED THREE AND SIX MONTHS IN ADVANCE[a] TRADE AND MANUFACTURING, 1948-1960
(Billions of dollars)

End of Quarter	Actual	Calculated 3 Months in Advance	Calculated 6 Months in Advance
1948 – 1	52.3	n.a.	n.a.
2	53.6	53.6	n.a.
3	55.3	54.9	55.2
4	55.9	55.6	55.7
1949 – 1	55.7	55.3	55.9
2	54.0	54.3	54.7
3	53.5	52.5	53.3
4	52.3	53.4	52.2
1950 – 1	53.1	52.6	53.9
2	54.8	54.6	54.0
3	57.8	58.4	57.8
4	63.4	63.0	62.7
1951 – 1	68.4	67.8	68.5
2	71.9	71.8	70.3
3	73.2	72.3	73.6
4	73.8	73.3	72.8
1952 – 1	74.5	74.2	73.0
2	73.3	74.7	74.0
3	73.8	74.5	74.4
4	75.4	75.6	75.1
1953 – 1	76.4	76.8	77.2
2	78.0	78.0	78.2
3	79.6	78.8	79.4
4	78.6	78.6	78.9
1954 – 1	77.6	77.0	77.6
2	76.7	76.4	75.4
3	75.4	76.1	75.4
4	75.5	75.1	75.7
1955 – 1	76.4	76.4	75.0
2	77.8	77.7	77.5
3	79.2	79.5	79.5
4	81.7	81.5	81.6
1956 – 1	83.7	84.3	84.1
2	85.6	85.4	87.2
3	86.9	86.8	86.6
4	89.1	88.8	88.3
1957 – 1	89.9	90.7	90.4
2	90.7	90.0	92.1
3	91.3	90.9	90.1
4	90.7	90.6	91.0
1958 – 1	88.5	89.3	89.8
2	86.4	86.6	87.9
3	85.0	86.1	84.7
4	85.1	85.6	86.2
1959 – 1	86.6	86.4	86.3
2	89.3	88.7	88.4
3	89.2	91.0	90.9
4	89.4	90.3	93.2
1960 – 1p	92.2	90.8	91.9

[a]The calculated level of inventories at the end of a forecast period equals the actual level at the beginning of that period plus the dollar change for the period derived from the forecasting formula less the inventory valuation adjustment for the period

na = Not available p = Preliminary

Sources: U.S. Department of Commerce; The Conference Board

Another possible use of the present results lies in forecasting the inventory change component of the gross national product account. Chart 23 shows the actual quarterly changes in nonfarm inventories in billions of current dollars, and the corresponding calculated changes in trade and manufacturing inventories.[1]

Techniques analogous to those used here to forecast total business inventories probably stand a good chance of success in application to particular industries, and perhaps also to specific types of inventory stocks. A forecast of change in total business inventories might also be constructed in detail from separate forecasts for the parts of the total.

The most important result of the present findings is that it seems practical to calculate inventory changes three and six months in advance, given the knowledge about the current levels of inventories and of unfilled orders and the current rate of sales and new orders. As long as the underlying conditions remain essentially as they were in the postwar period, the formulas constructed here, which were based on these conditions, should continue to produce useful measurements of current inventory tendencies, although, as with all predictions, they can hardly be taken as conclusive. The fact that a good explanation of inventory changes over a stretch of almost a dozen years has been obtained suggests that major postwar influences on inventory behavior have been covered by the analysis.

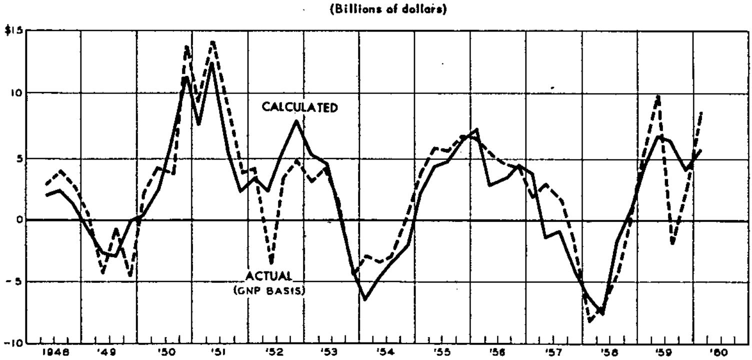
Eventually, of course, the underlying conditions will shift, the relative importance of the variables included in the analysis may change as far as their influence on inventory formation is concerned, and new significant influences may appear. (For example, a higher significance may accrue to interest rates and price expectations than appears in the postwar record.) And the techniques developed in this paper are not intended to deal with extraordinary circumstances, such as a serious strike. In such cases, the inventory changes indicated by the present analysis may not furnish a reliable measurement of inventory conditions, particularly with respect to the timing of inventory accumulation. However, they may be used as a guide to the degree of distortion created by unusual developments. In fact, a major use of the predictive equations developed here is as a guide in revealing temporary or long-term shifts in the influences affecting inventory conditions.

[1] In the postwar years, the total change in trade and manufacturing inventories accounted for 93% of the change of all nonfarm inventories. The rest was taken up by other nonfarm industries. The present forecast might be further refined by including an estimate of the prospective inventory change in these industries

Chart 23

Net changes in business inventories: changes calculated three months in advance and actual changes,▲ 1948-1960, quarterly, seasonally adjusted annual rates.

(Billions of dollars)



▲The actual changes are for total nonfarm sector; the calculated changes are for trade and manufacturing

Sources: Department of Commerce; The Conference Board